## MATHEMATICS ENRICHMENT CLUB.
### Problem Sheet 16, September 10, 2018

1. At a party of 11 people, everyone claims they shook hands with exactly 5 other people. Show that someone is lying.

2. The numbers on opposite faces of a standard 6 sided die always add up to 7. Suppose I have 27 standard dice arranged in a $3 \times 3 \times 3$ cube. What is the smallest possible sum of all the exposed faces?

3. A permutation is an ordered arrangement of objects. For example, 3124 is a permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically (or alphabetically, in the case of permutations of letters), we call it lexicographic order. For example, the permutations of 0, 1 and 2 in lexicographic order are:
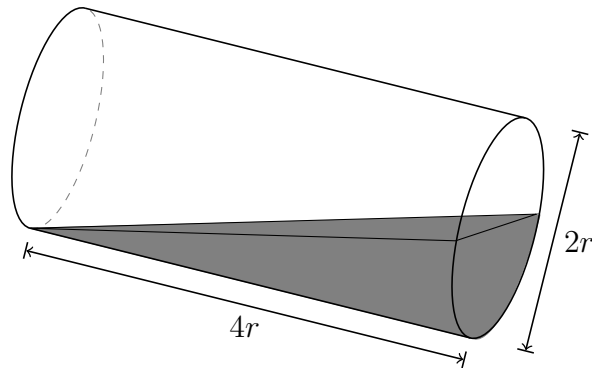
$$012, 021, 102, 120, 201, 210.$$

   What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9?

4. A $n$ digit number is narcissistic if the sum of its digits each raised to the power of $n$ equals the number. Show that there are no 2-digit narcissistic numbers.

5. **Construction problem:** Construct a right-angled triangle, given:

   (a) the hypotenuse and the sum of the two short sides.

   (b) the hypotenuse and the difference of the two short sides.[1]

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[1] Adapted from AP Kiselev *Kiselev's Geometry: Planimetry*, Tr. A Givental, 2006

**Senior Questions**

1. A regular polygon with n sides is inscribed in a circle. If $A$, $B$, $C$ and $D$ are four successive vertices of the polygon then the length of $AD$ equals the side of the polygon plus the radius of the circle. Find all possible values of $n$.

2. A glass has the shape of a right circular cylinder with base of radius $r$ and height twice the diameter of the base. The glass is tilted and partially filled with water so that the water touches the rim and covers half of the base, as shown in the diagram.



Calculate the volume of water in terms of $r$.