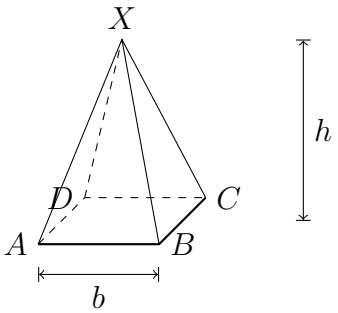
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 17, September 17, 2018

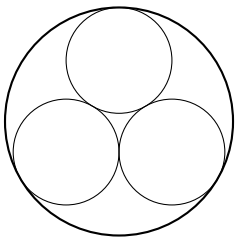
1. Suppose that
$$N = 1^9 \times 2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9^1.$$
How many perfect squares divide N?

2. Let $-10 \leq a, b, c \leq 10$. How many triplets, (a, b, c), satisfy
$$\frac{a/b}{c} = \frac{a}{b/c}?$$

3. A right square-based pyramid is placed on a table. The pyramid has base $ABCD$ with sides of length b and apex X at a height h above the base. What is the shortest distance an ant on the table can travel when moving from A to C?



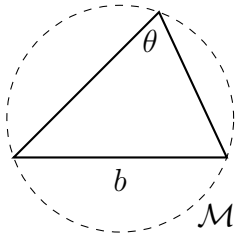
4. Three small circles with radius r are inscribed in a larger circle with radius R as shown in the diagram.



What is the relationship between r and R?

5. **Contruction Problem**

   (a) Show that you can construct the circumcircle, $\mathcal{M}$, of a triangle given the length of the base, b, and the angle at the apex, θ.



   (b) Construct a triangle, given the angle at the vertex; the length of the altitude from the vertex to the base; and the length of the median from the vertex to the base.

Senior Questions

1. A napkin ring is formed by drilling a hole of length h through the centre of a sphere of radius r. Find the volume of the napkin ring.

2. (a) Show that, for $-1 < x < 1$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots,$$

   and hence show that

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots.$$

   (b) Using the result from part (a), how many terms are needed to approximate $\ln(1.1)$ correct to 5 decimal places?