



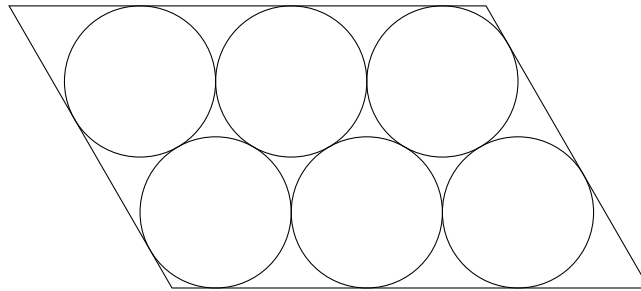
### MATHEMATICS ENRICHMENT CLUB.

#### Problem Sheet 3, May 21, 2018

1. The perimeter of a base of a rectangular brick with integer sides is 18 cm, whilst its volume is  $42 \text{ cm}^3$ . What is its height?
2. A palindromic number is a 'symmetrical' number which reads the same forwards as backwards. For example, 55, 101 and 8668 are palindromic numbers. There are 90 four-digit palindromic numbers.

How many of these four-digit palindromic numbers are divisible by 7?  
(AMC 2009 Senior Division Q21)

3. What is the area, in square centimetres, of the parallelogram that would fit snugly around 6 circles, each of radius 3 cm, as shown in the diagram?



(AMC 2009 Senior Division Q22)

4. Suppose that a triangle has sides  $a$ ,  $b$  and  $c$  such that  $a + b + c = 2$ .
  - (a) Show that  $(1 - a)(1 - b)(1 - c) > 0$ .
  - (b) Show that  $a^2 + b^2 + c^2 + 2abc < 2$ .
5. Suppose we have a sequence defined by  $x_0 = 0$ ,  $x_1 = 1$  and  $x_{n+1} = x_n + 2x_{n-1}$  for  $n \geq 2$ .
  - (a) Find the first five terms in the sequence.
  - (b) Show that  $x_n = \frac{2^n - (-1)^n}{3}$  satisfies the recurrence.
  - (c) Show that there is no  $n$  for which  $x_n = 2018$ .

### Senior Questions

1. Suppose we expand  $(3 + 2x + x^2)^{2018}$  to obtain  $a_0 + a_1x + a_2x^2 + \dots + a_{4036}x^{4036}$ .
  - (a) Find  $a_0$  and  $a_1$ .
  - (b) Find  $a_0 + a_1 + a_2 + \dots + a_{4036}$
  - (c) Find  $a_0 - a_1 + a_2 - a_3 + \dots + a_{4036}$
2. A trapezium  $ABCD$  has  $AD \parallel BC$  and a point  $E$  is chosen on the base  $AD$  so that the line segments  $BE$  and  $CE$  divide the trapezium into three right-angled triangles. These three triangles are similar, but no two are congruent. In common units, all the triangle's short side lengths are integers. The length of  $AD$  is 2009. What is the length of  $BC$ ? (AMC 2009 Senior Division Q30)