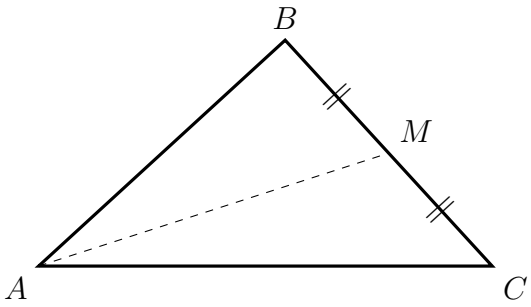
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 9, July 2, 2018

1. The angles in a triangle are in the ratio $2 : 3 : 4$. Find, in degrees, the size of the largest angle.

2. How many digits does the number 125^{100} have?

3. Let ABC be a triangle with AM one of its medians.



   Prove that AM is smaller than the semi-perimeter of $\triangle ABC$. That is, show that $AM < \frac{1}{2}(AB + BC + AC)$[1].

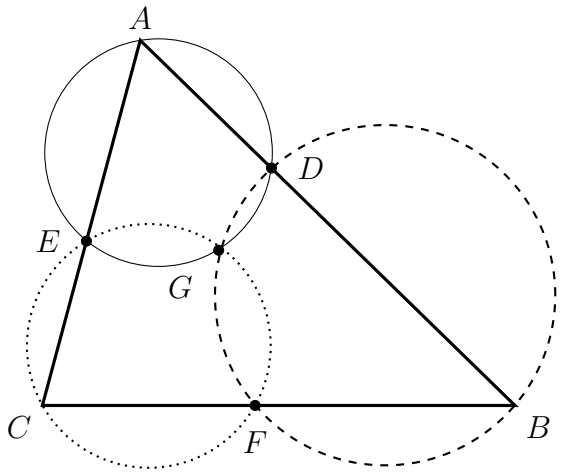
4. Let
$$\alpha = \frac{1}{1 + \frac{1}{1+\frac{1}{1+\ldots}}}.$$
   Evaluate α.

5. (a) Find the greatest common divisor of $2^{50} + 1$ and $2^{20} + 1$.
   (b) Explain why the greatest common divisor of $2^m + 1$ and $2^n + 1$ is at least three if m and n are both odd.

[1]This question is adapted from A. P. Kiselev, *Geometry: Planimetry*, tr. A Givental, 2006

Senior Questions

1. **The Miquel Point.** Let ABC be a triangle. Let D, E and F be points on the sides of the triangle. Show that circles through ADE, BDF and CEF intersect at a common point G as shown. [2]



2. By considering the equation $\cos(A+B)+\sin(A-B)=0$ find the general solution (for θ) of

$$\cos(n\theta)+\sin(m\theta)=0.$$

[2]This question is adapted from R. Hartshorne, *Geometry: Euclid and Beyond*, p 61