MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 1, May 6, 2019

1. Express 0.284284284… as a fraction in simplest terms.

2. Let $x$, $y$ and $z$ be integers. Show that if $x - y + 2z$ is divisible by 11, then so is $-12x + y - 13z$.

3. Anna and Boris move simultaneously towards each other from points $A$ and $B$, respectively. Their speeds are constant, but not necessarily equal. Had Anna started 30 minutes earlier, they would have met 2 kilometres nearer to $B$. Had Boris started 30 minutes earlier instead, they would have met $d$ kilometers nearer to $A$. Find $d$.

4. A four digit number and its square ends in the same four digits. Find the number.

5. A $3 \times 3$ magic square is a grid filled with the numbers 1 to 9 so that the sum of rows, column and diagonal are all equal. E.g

\[
\begin{array}{ccc}
6 & 1 & 8 \\
7 & 5 & 3 \\
2 & 9 & 4 \\
\end{array}
\]

Counting different orientations of the grid as the same magic square, prove that the above example is the only solution.
Senior Questions

1.  (a) Prove the identity

\[ \frac{d}{dx} \tan^{-1}(x) = \frac{1}{1 + x^2}. \]

(b) Using the this result, show that the infinite series satisfies

\[ x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots = \tan^{-1}(x). \]

2.  (a) For an integer \( n \), show that \( n(n + 1)(n + 2)(n + 3) + 1 \) is a perfect square.

(b) Thus evaluate \( \sqrt{(31)(30)(29)(28) + 1}. \)