MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 12, August 20, 2019

1. Let $n!$ denote the factorial of n. That is,

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1.$$

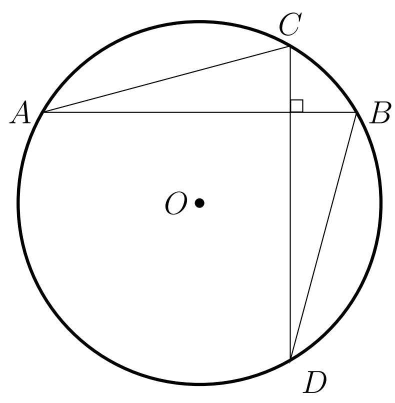
   Find the largest integer n, such that $1 + 2! + 3! + \ldots + (n-1)! + n!$ is a perfect square.

2. Let $a_1, a_2, \ldots, a_{100}$ be a sequence of consecutive positive integers. Find the minimum value of $\sqrt{a_2 + a_3 + \ldots + a_{99}} - \sqrt{a_1 + a_{100}}$.

3. A 6-digit number is increased 6 times when its last 3 digits are carried to the beginning of the number without their order being changed. Find this number.

4. A large number of brown, green and yellow frogs are wandering around on an island. Whenever two frogs of different colours meet each other, they change immediately into two frogs of the third colour. More than two frogs never meet simultaneously. If there are initially 2017 brown frogs, 2018 green frogs and 2019 yellow frogs on the island, is it possible that at some future time all the frogs will have the same colour?

5. A circle with radius R is centered at O. Suppose that AB and CD are perpendicular chords in the circle, as shown in the diagram.

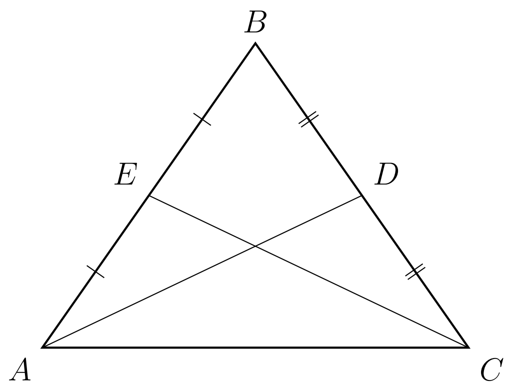


   Show that

$$AC^2 + BD^2 = 4R^2.$$

Senior Questions

1. Let ABC be a triangle. Let D and E be the feet of the medians from A to BC and B to AC, respectively.



Prove that if $CE = AD$ then $\triangle ABC$ is isosceles.

2. Express

$$\sum_{n=1}^{25} \frac{2n-1}{n(n+1)(n+2)}$$

as a fraction of two co-prime positive integers.