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MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 16, September 17, 2019

1. Suppose that a is a real number such that $0 < a < 1$. List the following numbers in ascending order: a^3, a^2, a, $-a$ and $\sqrt{a}$.

2. Is it possible to cut a square into nine squares and colour one of them white, three of them gray and five of them black, such that squares of the same colour have the same size and squares of different colours have different sizes?

3. *AMC 2010 Senior Division, Q26.*
If $m + n = 11$ and $m^2 + n^2 = 99$, what is the value of $m^3 + n^3$?

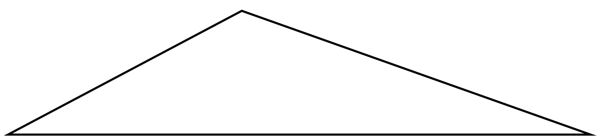
4. *AMC 2012 Senior Division, Q23.*
If x and y are positive integers which satisfy $x^2 - 8x - 1001y^2 = 0$, what is the smallest possible value of $x + y$?

5. *AMC 2012 Senior Division, Q25.*
In 'base -2' notation, digits are 0 and 1 only and the places go up in powers of -2. For example, 110110 stands for

$$1 \times (-2)^5 + 1 \times (-2)^4 + 0 \times (-2)^3 + 1 \times (-2)^2 + 1 \times (-2)^1 + 0 \times 1 = -14.$$

If the decimal number 2000 is written in base -2 notation, how many non-zero digits does it contain?

6. *AMC 2010 Senior Division, Q15.*
The length of each side of a triangle such as the one below is a different prime number and its perimeter is also a prime number.



What is the smallest possible perimeter of such a triangle?

Senior Questions

1. Billy summed up 10 consecutive powers of 2, starting from some power, while Joe summed up several consecutive positive integers starting from 1. Can they get the same results?

2. Evaluate
$$\lim_{n\to\infty} \sqrt[n]{\frac{(2n)!}{n!n^n}}.$$
Hint: use calculus.

3. Let x and y be real numbers satisfying $x^4y^5 + y^4x^5 = 810$ and $x^3y^6 + y^3x^6 = 945$. Evaluate $2x^3 + (xy)^3 + 2y^3$.