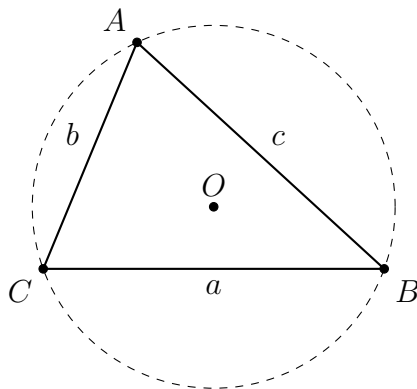


MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 2, May 13, 2019¹

1. The diagonal of a polygon is a line joining two non-adjacent vertices. For example, the two diagonals of a square. A polygon has 152 diagonals, how many vertices does it have?
2. Find all primes p such that $17p + 1$ is a square.
3. A politician counted that the total number of staff working for him is the product of two consecutive numbers. In an upcoming election campaign, the politician plans to visit the schools in a city. The politician decides to split all his staff, himself and his wife into groups of three to make the visits, explain to the politician why this is mathematically impossible.
4. Each of six baskets contains some mangoes, peaches and apples. The number of peaches in each basket is equal to the total number of apples in the other five baskets, and the number of apples in each basket is equal to the total number of mangoes in the other five baskets. Prove that the total number of fruit in the six baskets is a multiple of 31.
5. The diagram below shows $\triangle ABC$ and its circumcircle (the circle that passes through all three vertices of the triangle). The centre of the circumcircle is O and its diameter is d .



- (a) Show that

$$\frac{a}{\sin A} = d.$$

- (b) Hence derive the sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

¹Some problems are from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*.

Senior Questions

1. Alex has a piece of cheese. He chooses a positive number $x > 1$ and cut the piece into two, in the ratio $1 : x$. He can then choose any piece and cut it in the same way. Is it possible for him to obtain, after a finite number of cuts, two piles of pieces each containing half the original amount of cheese?

2. Find a polynomial function $f(x)$ which has its inverse equal to its derivative, that is,

$$f^{-1}(x) = f'(x).$$

3. (a) Find the next 3 terms of the sequence $1, 1, 2, 3, 5, 8, 13, \dots$
(b) Let F_n be the n th term of the above sequence. Prove that the sum of the sequence is $S_n = F_{n+2} - 1$.