Never Stand Still | Science | School of Mathematics and Statistics

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 8, June 24, 2019

1. Given that x and y are integers, how many different solutions are there to the equation

$$|x| + 2|y| = 100?$$

2. Place the numbers $1, 2, 4, 8, 16, 32, 64, 128$ and 256 in a 3×3 square grid in such a way, that the product of each row, column and diagonal gives the same value. Can you find another solution that is not just a rotation or reflection of the first one?

3. If x is a positive real number, let $[x]$ denote the greatest integer less than or equal to x and $\{x\} = x - [x]$. For example, $[3.14] = 3$, $\{3.14\} = 0.14$.

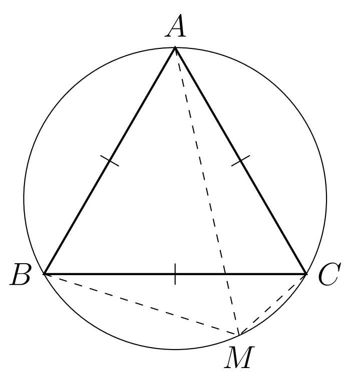
   Find numbers x and y such that

   (a) $x^3 - 5[x] = 10$

   (b) $y^3 - 5\{y\} = 10$.

4. Consider the polynomial $f(x) = x^4 - nx + 63$. Find the smallest positive integer n such that $f(x)$ can be written as the product of two non-constant polynomials with integer coefficients.

5. Let $\triangle ABC$ be an equilateral triangle inscribed in a circle. Let M be fourth point on the circle, lying between B and C, as shown below. Prove that $MA = MB + MC$.



Senior Questions

1. How many different integers x satisfy the equation

$$(x^2 - 5x + 5)^{x^2-11x+30} = 1?$$

2. The integers 5, 11, 17, 23 and 29 are five prime numbers in arithmetic progression. Find *six* prime numbers in arithmetic progression.

3. Given that n is a positive integer and $2n+1$ and $3n+1$ are perfect squares, prove that n is divisible by 40.