Purchasing Power Parities, Price Levels and Measures of Regional and Global Inflation

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Outline

- **Key Concepts and notation**
  - **Purchasing power parities (PPPs)**
  - **Price Levels**
- **Temporal-spatial comparisons**
  - **GEKS Approach with Fixity**
- **Comparisons of World GDP over time**
  - **Global Inflation**
  - **Global Growth**

Key Concepts and Notation

Gross Domestic Product of country j

(in national currency units) $GDP^j = \sum p_n^j \cdot x_n^j$

Nominal GDP : GDP of country i in period t expressed in reference currency units using exchange rates

$$NGDP^j = \frac{GDP^j}{XR^j}$$

Real GDP : GDP of country i expressed in reference currency units converted using PPPs

$$RGDP^j = \frac{GDP^j}{PPP^j}$$

Purchasing Power Parities (PPPs)

PPPs are amounts of currencies, of different countries, that have the same purchasing power as one unit of a reference currency (e.g. US$) with respect to a selected basket of goods and services.

PPPs from ICP 2005

Country	Exch. Rate	PPP	PLI% (US=100)
P.R. China	8.19	3.56	43
Hong Kong	7.78	5.86	75
India	44.10	15.15	34
Australia	1.31	1.39	106
Japan	110.22	129.55	118
Switzerland	1.25	1.75	140
Ethiopia	8.67	2.33	27

Source: World Bank, 2005 ICP Report

Price Level Indices

Price Level Index (PLI): It is defined as the ratio of PPP to the exchange rate

$$PLI^{1j} = \frac{PPP^{1j}}{XR^{1j}}$$

Properties of PLIs:

- **By definition PLI of reference country is 1**
- **Interpretation of PLI can be ambiguous**
- **PLIs for a country using two different reference countries are not directly comparable**
- **PLIs are transitive**

Price Level Indices

Price Level Index (PLI): It is defined as the ratio of PPP to the exchange rate

$$PLI^{1j} = \frac{PPP^{1j}}{XR^{1j}} = \frac{GDP^{j} / XR^{1j}}{GDP^{j} / PPP^{1j}}$$

Properties of PLIs:

- **By definition PLI of reference country is 1. For this country the nominal and real GDP are the same**
- **Interpretation of PLI can be ambiguous**
- **PLIs for a country using two different reference countries are not directly comparable**
- **PLIs are transitive**

Price Level Indices Relative to World Average

1. **A simple approach is to compute the geometric average of all PLI's and normalise PLI of each country using the geometric mean.**
   - **Such a normalisation does not have any useful interpretation.**
2. **An alternative is to compute the weighted average of PLIs and use a normalisation based on the average.**

$$\text{World average of } PLIs = \mu^1 = \sum_{j=1}^{I} PLI^{1j} \frac{RGDP^{1j}}{\sum_{j=1}^{I} RGDP^{1j}}$$

3. **Scale the PPPs to the world average as:**

$$PPP^{Wj} = PPP^{1j} / \mu^1$$

Price Level Indices Relative to World Average

4. **PLI for country *j* with world average is:**

$$PLI^{Wj} = PPP^{Wj} \Big/ XR^{1j}$$

5. **World average of PLI's at new PPPs is equal to 1 and, therefore:**

$$\sum_{j=1}^{I} GDP^{j} \Big/ PPP^{Wj} = \sum_{j=1}^{I} GDP^{j} \Big/ XR^{1j}$$

World GDP at the new PPPs is equal to world nominal GDP at exchange rate

6. **PLI's defined using *world average* are invariant to the choice of the reference country for PPPs.**

Table 1: Price Level Indexes for Selected Countries, 2005
(Price level in USA = 1.0)

Country	PPP	XR	PLI with USA = 1	PLI with world average. =1
Australia	1.39	1.31	1.06	1.32
Denmark	8.52	5.95	1.42	1.76
India	14.67	44.10	0.33	0.41
Brazil	1.36	2.43	0.56	0.69
South Africa	3.87	6.36	0.61	0.76
USA	1	1	1	1.24

Table 2: PPPs and Real Incomes for Selected Asia-Pacific Countries, 2005 and 2009
Reference Currency – Hong Kong dollar

Country	2005					2009			
	PPP	XR	PLI*	Real per capita GDP		PPP	XR	PLI*	Real per capita GDP
Bangladesh	3.98	8.27	86	7,215		4.16	8.91	76	10,888
China, PR	0.61	1.05	103	23,938		0.64	0.88	117	40,706
Hong Kong	1	1	180	202,941		1	1	162	231,611
India	2.58	5.67	82	12,090		2.70	6.25	70	19,500
Malaysia	0.31	0.49	112	65,217		0.29	0.45	104	82,524
Thailand	2.80	5.17	97	39,070		2.47	4.42	91	52,753
Vietnam	829	2039.12	73	12,185		950.68	2201.95	70	20,389

Note: (*) PLIs are defined relative to regional average with Asia = 100.
Source: ADB (2007) and ADB (2012)

GEKS Space-Time Comparisons

The Problem:

- **Consider the case of two periods and comparisons involving M countries.**
- **Suppose we have ICP results for the two periods. We have PPPs for the two periods, denoted by** PPP_{jk}^{tt} **for** t **=1,2.**
- **Let the results for periods 1 and 2 be represented by matrices:** $\prod^{11}$ and $\prod^{22}$

- **Since PPPs are transitive, elements of these matrices can be written as, for** t**=1,2**

$$PPP_{jk}^{tt} = \frac{\pi_k^t}{\pi_j^t}$$

GEKS Space-Time Comparisons

The Problem:

- **We wish to construct a transitive matrix which allows comparisons between periods 1 and 2.**
- **The problem is to fill the elements of the matrix, Π of order (2M x 2M) of the form:**

$$\Pi = \begin{pmatrix} \Pi^{11} & \Pi^{12} \\ \Pi^{21} & \Pi^{22} \end{pmatrix}$$

- **Note that Π^{11} and Π^{22} are given from the ICP.**
- **Typical element of Π^{12} are of the form PPP_{jk}^{12} - PPP for country *k* in period 2 with country *j* in period 1 as the reference.**

GEKS Space-Time Comparisons

The approach:

- **Fill the elements of the off-diagonal matrices, Π^{12} and Π^{21} using observed domestic deflators in the M countries over the period 1 to 2.**
- **We have the following choices:**

$$PPP_{jk}^{12} = PPP_{jk}^{11} \cdot d_k^{12}$$

$$PPP_{jk}^{12} = PPP_{jk}^{22} \,/\, d_j^{21}$$

- **Since both of these are equally desirable, we use the geometric mean.**

$$PPP_{jk}^{12} = \left[\left(PPP_{jk}^{11} \cdot d_k^{12}\right)\left(PPP_{jk}^{22} \,/\, d_j^{21}\right)\right]^{1/2}$$

- **We note that the matrix Π with the observed Π^{11} and Π^{22} and filled in Π^{12} and Π^{21} is not transitive.**
- **We apply GEKS to this matrix.**

GEKS Space-Time Comparisons

The approach (continued)

- We note that straightforward application of GEKS will disturb the PPP relativities in periods 1 and 2 as obtained from the ICP.
- So, we apply GEKS imposing temporal fixity. Accordingly we minimise

$$\sum_{t=1}^{T}\sum_{s=1}^{T}\sum_{j=1}^{M}\sum_{k=1}^{M}\left[\ln PPP_{jk}^{ts}-\ln \pi^{*s}_{k}+\ln \pi^{*t}_{j}\right]^{2}$$

subject to fixity restrictions:

$$\pi^{*s}_{k}=\delta^{*s}\,\pi^{s}_{k} \text{ for } k=1,2,\ldots,M \text{ and } s=1,2.$$

- The solution to GEKS with fixity is:

$$\delta^{*1}=1 \textit{ and } \delta^{*2}=\prod_{j=1}^{M}\left[d_{j}^{12}\right]^{1/23}$$

GEKS Space-Time Comparisons

In the context of Asia-Pacific Example our results suggest to link 2005 with 2009 using:

$$\delta^{*2} = \prod_{i=1}^{23} \left[d_i^{2005,2009} \right]^{1/23}$$

a geometric mean of price movements from 2005 to 2009 in each of the 23 countries.

δ_2 can be interpreted as a measure of *regional inflation*.

However, use of unweighted average of country specific inflations is not satisfactory. If we want to use weights, which weights do we use?

Regional/Global Deflator

Using the *plutocratic* structure of the regional PLI, we may define regional deflator as:

$$\delta^{*2} = \prod_{i=1}^{23} \left[d_i^{2005,2009} \right]^{w_i}$$

where

$$w_i = \frac{w_i^{2005} + w_i^{2009}}{2} \text{ with } w_i^t = \frac{RGDP_i^t}{\sum_{i=1}^{23} RGDP_i^t}$$

δ_2^* **is like a Tornqvist index of price movements from 2005 to 2009.**

Measures of Global Inflation and Global Growth

Motivated by the result of GEKS with fixity, we proceed to obtain measures of *global inflation* and *global growth*.

We use the following approach:

- **We first define global GDP in periods *s* and *t* which are given by**

$$RGDP^s = \sum_{j=1}^{M} RGDP_j^s = \sum_{j=1}^{M} GDP_j^s / PPP_j^s$$

$$RGDP^t = \sum_{j=1}^{M} RGDP_j^t = \sum_{j=1}^{M} GDP_j^t / PPP_j^t$$

- **We note that these two aggregates are in the prices of periods *s* and t respectively. The ratio of these two aggregates is similar to the ratio of nominal GDP in two periods.**

Measures of Global Inflation and Global Growth

- **The nominal growth in world GDP for period *s* to *t* is given by:**

$$\frac{\sum_{j=1}^{M} GDP_j^t / PPP_j^t}{\sum_{j=1}^{M} GDP_j^s / PPP_j^s}$$

- **The objective is to decompose this ratio as:**

Nominal growth in World GDP = WI(s,t)*WG(s,t)

- **We provide expressions for WI(s,t) and WG(s,t)**

Measures of Global Inflation and Global Growth

- **We first observe that**

$$\frac{\sum_{j=1}^{M} GDP_j^t / PPP_j^t}{\sum_{j=1}^{M} GDP_j^s / PPP_j^s} = \exp\left\{\sum_{j=1}^{M} \Psi^j \ln\left[\frac{GDP_j^t / PPP_j^t}{GDP_j^s / PPP_j^s}\right]\right\}$$

$$= \exp\left\{\sum_{j=1}^{M} \Psi^j \ln\left[\frac{RGDP_j^t}{RGDP_j^s}\right]\right\}$$

where

$$\Psi^j = \frac{L\left[\frac{RGDP_j^s}{\sum_{k=1}^{M} RGDP_k^s}, \frac{RGDP_j^t}{\sum_{k=1}^{M} RGDP_k^t}\right]}{\sum_{j=1}^{M} L(.,.)} \quad \text{where } L(a,b) = \frac{a-b}{\ln a - \ln b} \text{ is the logmean.}$$

Measures of Global Inflation and Global Growth

- **Second, we decompose growth nominal GDP in country *j* into price and quantity components.**

$$\frac{GDP_j^t}{GDP_j^s} = P_j^{GDP}(s,t) \cdot Q_j^{GDP}(s,t) \quad \forall j$$

- **Then the World Inflation and World growth are measured by:**

$$WI(s,t) = \exp\left\{\sum_{j=1}^{M} \Psi^j \ln\left(P_j^{GDP}(s,t) \cdot \frac{PPP_j^s}{PPP_j^t}\right)\right\} \quad WG(s,t) = \exp\left\{\sum_{j=1}^{M} \Psi^j \ln\left(Q_j^{GDP}(s,t)\right)\right\}$$

- **It is quite easy to check that:**

$$\text{Nominal growth in World GDP} = \frac{\sum_{j=1}^{M} GDP_j^t / PPP_j^t}{\sum_{j=1}^{M} GDP_j^s / PPP_j^s} = WI(s,t) \cdot WG(s,t)$$

Empirical Application

Economy/country	2005				2009			
	Purchasing Power Parity (PPP)	Exchange Rate (LCU/HK$)	Price Level Index (Asia = 100)	Real GDP (million)	Purchasing Power Parity (PPP)	Exchange Rate (LCU/HK$)	Price Level Index (Asia = 100)	Real GDP (million)
Bangladesh	3.98	8.27	86	988332	4.16	8.91	76	1570115
Bhutan	2.77	5.67	88	13340	2.53	6.25	66	24213
Brunei Darussalam	0.21	0.21	133	99916	0.14	0.19	121	111994
Cambodia	225	526.21	77	114297	229.74	534.11	70	188422
China, People's Republic	0.61	1.05	103	30334238	0.64	0.88	117	54331621
Fiji	0.25	0.22	288	20167	0.26	0.25	169	21133
Hong Kong, China	1	1	180	1382675	1	1	162	1622203
India	2.58	5.67	82	13315076	2.7	6.25	70	22735794
Indonesia	692	1247.82	100	4026228	730.83	1340.63	89	7667814
Lao PDR	525	1370.03	69	58230	473.18	1098.84	70	100517
Malaysia	0.31	0.49	112	1703958	0.29	0.45	104	2336258
Maldives	1.43	1.65	156	6711	1.42	1.65	139	13297
Mongolia	73.4	154.97	85	38306	94.5	185.52	83	69510
Nepal	3.98	9.18	78	155766	4.71	10.01	76	228090
Pakistan	3.36	7.65	79	2098218	4.15	10.54	64	3319126
Philippines	3.82	7.08	97	1421731	3.58	6.15	95	2240383
Singapore	0.19	0.21	159	1024330	0.18	0.19	155	1498403
Sri Lanka	6.18	12.92	86	389389	7.99	14.83	87	604041
Taipei, China	3.4	4.14	148	3358809	2.98	4.26	113	4190945
Thailand	2.8	5.17	97	2530303	2.47	4.42	91	3663144
Vietnam	829	2039.12	73	1012850	950.68	2201.95	70	1753993
			RGDP(2005)	**64092870**			**RGDP(2009)**	**108291016**

Empirical Application

country	Deflators 2005 to 2009	2005	2009	Psi
Bangladesh	1.301482	0.015420311	0.014499	0.01497
Bhutan	1.204442	0.000208135	0.000224	0.000216
Brunei Darussalam	0.977021	0.001558925	0.001034	0.00128
Cambodia	1.282412	0.001783303	0.00174	0.001763
China	1.196693	0.473285687	0.501719	0.487855
Fiji	1.100538	0.000314653	0.000195	0.00025
Hong Kong SAR, China	1.035104	0.021572992	0.01498	0.018095
India	1.297166	0.207746603	0.209951	0.209057
Indonesia	1.623795	0.062818657	0.070807	0.066801
Lao PDR	1.257989	0.000908525	0.000928	0.000919
Malaysia	1.131726	0.026585765	0.021574	0.024017
Maldives	1.398922	0.000104707	0.000123	0.000114
Mongolia	1.684003	0.000597664	0.000642	0.00062
Nepal	1.414327	0.002430317	0.002106	0.002267
Pakistan	1.65756	0.032737152	0.03065	0.031714
Philippines	1.195867	0.022182358	0.020689	0.021448
Singapore	1.100559	0.015981965	0.013837	0.014899
Sri Lanka	1.562833	0.006075387	0.005578	0.005829
Taipei	0.961215	0.052405346	0.038701	0.045253
Thailand	1.153598	0.039478697	0.033827	0.036617
Vietnam	1.503539	0.01580285	0.016197	0.016015

Growth in nominal GDP in Asia = 1.68595

Asian Inflation = 1.2025

Asian Growth = 1.405

Thank you!