Honours in Applied Mathematics T3 2021:

Presentation schedule and abstracts.

DATE	TIME	STUDENT	TITLE (updated)	Supervisor(s)
Mon 15 Nov: 9-10AM	9:00	Jordan Lorenzatto	Exploring Stratosphere-Troposphere Coupling and its role in Southern Hemisphere Surface Climate	Martin Juker; co - Jan Zika
	9:20	Nicholas Killeen	Eigenfunction localisation of the Schrödinger operator for disordered anisotropic potentials	Bill McLean
	9:40	Brock Sherlock	Viruses as a Treatment for Cancer: Modelling Therapy Efficacy	Adelle Coster and Quoc Thong Le Gia
Mon 15 Nov: 10-11AM	10:00	Liam Roozendaal	Mathematical Modelling of Drug Epidemics to Test Intervention Strategies	Adelle Coster, Don Weatherburn, Anna Cai
	10:20			
	10:40	Wilson Yin	Future Applications of the Board Tutorial in UNSW Mathematics	Chris Tisdell
Mon 15 Nov: 11AM-12PM	11:00	Harish Suresh	Distilling the ocean's role in climate using phase diagrams and machine learning	Jan Zika, Taimoor Sohail
	11:20	Callum Scott	Solving partial differential equations by neural networks and deep learning	Quoc Thong Le Gia
	11:40	Michelle Qiu	Machine Learning with Applications into Financial Market	Quoc Thong Le Gia
Tue 16 Nov: 10-11AM	10:00	Jack Buckley	Target Detection by Kelvin Wake Fingerprinting	Shane Keating
	10:20	Matt Pudig	The Impact of Oscillatory Forcing on Ocean Heat Uptake	Jan Zika and Ryan Holmes
	10:40	Kaia Reynolds-Erler	Oceanographic Modelling: impact of changes in Westerlies on ocean circulation	Laurie Menviel; A. Schaeffer
Tue 16 Nov: 11AM-12PM	11:00	Stuart-James Burney	Particle-Based Simulations Produce Spatiotemporal Patterns in Static and Growing Escherichia Coli Cells	Christopher Angstmann
	11:20	William Du	Modelling Lateral Diffusion on Cell Surfaces	Adelle Coster
	11:40	Yuan Yuan Wang	Application of the classical Moutard transformation to surfaces of constant negative Gaussian curvature	Wolfgang Schief
Tue 16 Nov: 3-4PM	15:00			
	15:20	Kabir Agrawal	Waves generated by moving bodies in two dimensions	Shane Keating
	15:40	Josef Bisits	Passive tracer mixing along isopycnal surfaces in turbulent flows	Jan Zika, Geoff Stanley
Tue 16 Nov: 4-5PM	16:00			
	16:20	Ming Hao	Laminar Flow in Channels with Porous Walls - with slip boundary conditions	Chistopher Tisdell
	16:40	Lesley Gao	Pattern Formation in Fractional Order Dynamical Systems	Christopher Angstmann, Bruce Henry
Tue 16 Nov: 5-6PM	17:00	Bruce Cable	Pseudorandom Properties of the Subset Maps and Markoff Hash Function	Igor Shparlinski

** Please note the times might slightly vary.

- Jordan Lorenzatto

Exploring Stratosphere-Troposphere Coupling and its role in Southern Hemisphere Surface Climate

This thesis aims to investigate the dynamical coupling of the Stratosphere and Troposphere in the Southern Hemisphere. The main focus is the Southern Annular Mode (SAM), a significant mode of climate variability that can be defined at all pressure levels throughout the atmosphere, and the analysis of how its anomalies propagate between the atmospheric layers. Previous studies have investigated this coupling by observing stratospheric annular mode variability and how it propagates downwards. In this thesis, I instead observe periods of tropospheric variability and examine their stratospheric precursors. Following this, I explore a possible dynamical explanation of how SAM anomalies propagate through wave-mean flow interaction, specifically the relative effects of vertical momentum flux. This explanatory effort proves to be inconclusive, however, indicating a need for either a similar analysis of more data or further research into the plausible dynamical causes.

- Nicholas Killeen

Eigenfunction localisation of the Schrödinger operator for disordered anisotropic potentials

Typical eigenstates of quantum systems with regular potentials are smooth, oscillatory functions with global support. However, when the potential energy is sufficiently disordered, a phenomenon called Anderson localisation can occur, in which an eigenstate is a spike occupying only a small region of the spatial domain. This phenomenon supports an enormous body of literature in various areas of applied mathematics, a key portion of which recently involves using the groundbreaking development of landscape localisation theory to compute estimates for the location and shapes of the Schrödinger operator's eigenstates and eigenvalues given particular realisations of random potentials, without having to solve computationally expensive eigenproblems. Study so far has been concerned with isotropic potentials, and it remains to be answered the conditions and character of localisation for more structured potentials. In this paper, we build up FEM architecture to explore the phenomenon of eigenstate localisation for random anisotropic potentials and assess the effectiveness of the new landscape localisation methodology for estimating low-energy spectral information for these potentials.

- Brock Sherlock

Viruses as a Treatment for Cancer: Modelling Therapy Efficacy

Oncolytic viruses a promising new treatment for cancer, whereby viruses are engineered to selectively destroy cancer cells. Some clinical trials have taken place using a variety of oncolytic viruses as well as several animal studies with encouraging results. However, available data is limited. The dynamics of the virus-tumour system can be modelled by a system of ODEs which provides insight into the system outcomes under treatment protocols not investigated by experiments. By modelling and simulating the system we acquire the capacity to predict patient outcomes and optimise the treatment for patient specific tumours. I will show that the optimal treatment options can be counter intuitive and how a framework has been developed to build an expert system to assess treatment options.

- Liam Roozendaal

Mathematical Modelling of Drug Epidemics to Test Intervention Strategies

Drug epidemics continue to pose challenges for governments as the intervention strategy which most effectively tackles drug use remains unclear. This thesis extends a dynamical system model of drug epidemics by better optimising its fit to the cocaine epidemic data in America (1971-1990), using the simplex method. With this revised mathematical model, intervention strategies are simulated at different times. From the simulations, connections are drawn between the effectiveness of the intervention strategy to the epidemic stage of implementation. Further, the model shows some seemingly positive strategies are counterintuitive and end up worsening the epidemic. The simulations also show the existence of a time lag for beneficial policies taking effect, which may make it seem like the epidemic is getting worse and the strategy is redundant. Additionally, new dynamical system models for drug epidemics will be introduced and examined.

- Wilson Yin

Future Applications of the Board Tutorial in UNSW Mathematics

The Board Tutorial is an innovative approach to tutorials currently used in MATH2601, where students work together on blackboards/whiteboards to solve problems.By breaking standard classroom norms, the Board Tutorial not only provides an Active Learning experience, but also encourages students to participate.However, the Board Tutorial has not translated particularly well to the online setting UNSW has found itself in for the past 2 years and may find continuing struggle if social distancing is required again.Despite this, proper implementation of the Board Tutorial across UNSW Mathematics can result in greater numbers of inquisitive mathematicians in the future.I will first present an overview of qualitative methods, explaining how I used interviews to gather the data I required and analysed them in a structured and replicable manner.Then, I will present my findings on the principals underpinning the Board Tutorial and its subsequent future applicability in UNSW.

- Harish Suresh

Distilling the ocean's role in climate using phase diagrams and machine learning

The global ocean exhibits complex circulations and structures which tend to organise into modes, or large, geographic regions where temperature and salinity (amongst other properties) remain relatively similar. Approximating the distribution of these modes is useful for data compression, computational efficiency, infilling sparse observational data, and objectively identifying dynamically similar regions to complement current, ad hoc methods of characterising ocean features such as fronts and water masses. Previous work has shown that unsupervised machine learning, specifically Gaussian Mixed Models (GMMs), can be used to classify vertical temperature profiles and produce spatially coherent clusters which discern structures known a priori within localised regions of the global ocean. This thesis extends upon past work by classifying both vertical temperature and salinity profiles globally, sourced from monthly-averaged gridded hydrographic observations between 2015 and 2019, to identify oceanographic structures and their evolution through time. Our analysis produces clusters with a high degree of certainty, that are both spatially and temporally coherent and correlate with known features. Further, sub-sampling from these clusters when calculating ocean heat content from 1970 onwards proved more stable than sub-sampling from basins, highlighting the utility of our clusters over geographic regions. This thesis provides a proof of concept for how classifying vertical temperature and salinity profiles provides objective clusters that are potentially more meaningful than traditional partitions. Future work could build upon this by using individual profiles rather than gridded observations, introducing more properties to classify, and applying these clusters to different problems, such as re-creating missing historical salinity profiles.

- Callum Scott

Solving partial differential equations by neural networks and deep learning

Partial differential equations (PDEs) on spheres have many applications in geosciences, but finding analytical solutions to many can be almost impossible. In this thesis, we investigate the use of machine learning techniques as numerical solvers for these PDEs with the intent of finding an efficient method which produces extensible solutions. Previous methods for finding numerical solutions to these equations have suffered from an inability to adapt solutions to different meshes, and contemporary machine learning methods have failed to highlight their applications on the sphere. We introduce a deep learning model that takes advantage of the backpropagation algorithm to calculate derivatives efficiently, and is able to produce results that are competitive with the spectral Galerkin method whilst avoiding the associated brittleness. We present numerical experiments demonstrating the performance of the model and compare and contrast these results with those achieved by the spectral Galerkin method. This opens up possibilities in optimisation studies with regards to PDEs on the sphere and removes the need for expensive generation of data sets that other deep learning models rely on. We believe these results have a use case in the natural sciences for which analytical solutions to PDEs are difficult to find.

- Michelle Qiu

Machine Learning with Applications into Financial Market
(Abstract not provided)

- Jack Buckley

Target Detection by Kelvin Wake Fingerprinting

The ability to detect an underwater object from the surface is a key objective of modern marine defence systems. Yet, the conventional method of submarine detection via sonar has become more difficult with the advancement of sound absorption technology. So, alternate methods of detection are of growing interest. Current target detection methods involve producing images simulating the object's wake and then analyzing the resulting wake pattern using, for example, the Fourier power spectrum \cite{xue2020wake}, a Radon transform \cite{zilman2014detectability} or a neural network \cite{fitch1991ship}. Researchers have also explored evaluating the electromagnetic fields produced by the wakes as a means of identifying the presence of a submarine \cite{xu2017evaluation}. In this thesis, we explore whether it is feasible to detect submerged objects from their surface wake patterns using modern Machine Learning (ML) techniques. Initially, we explore whether our method can identify the presence of surface objects amidst realistic environmental noise, since the magnitude of the wake pattern diminishes significantly as the depth of the body increases. Fundamentally, the results we obtain demonstrate the potential of ML algorithms for rapid and accurate detection of submerged objects.

- Matt Pudig

The Impact of Oscillatory Forcing on Ocean Heat Uptake

The ocean is the primary mediator of heat in Earth's climate system. However, the ocean uptakes heat nonlinearly, absorbing cooling more efficiently than warming at the sea surface. Because of this nonlinearity, oscillatory forcing can induce a long-term trend in ocean heat content. This has implications for how state-of-the-art climate models are forced and initialised, such as the impact of volcanic eruptions, and affects future projections of heat content change made with these models. To explore how nonlinear ocean heat uptake interacts with forcing variability, this study compares results from two models, an ocean general circulation model and an idealised 1D vertical diffusion model, both forced with idealised oscillatory surface heat fluxes. The 1D model shows qualitatively similar results to the general circulation model. Because of its cheap computational cost, the 1D model permits a more thorough exploration of the nonlinear responses, such as the sensitivity to the amplitude and period of the oscillatory forcing as well as how the response depends on the vertical diffusivity. This study sheds light on the physical mechanisms that might lead to uncertainty in heat content estimates made by climate models.

- Kaia Reynolds-Erler

Oceanographic Modelling: Impact of changes in Southern Hemisphere Westerlies on the AMOC

The Atlantic Meridional Overturning Circulation (AMOC) links North Atlantic Deep Water and Antarctic Bottom Water. The AMOC helps regulate sea levels, surface temperature and rainfall over the North Atlantic and Western Europe and is heavily influenced by the prevailing wind systems. The Southern Hemisphere (SH) Westerly Winds operate above the Southern Ocean, and regulates; key currents related to the AMOC and the volume of deeper waters upwelled into the thermocline of the Southern Ocean. Over the last 30 years, satellite measurements have recorded a 20% increase in the jet strength of the SH westerlies, yet the influence of this change on AMOC related circulation are poorly constrained. This project sets out to understand the impact that the measured changes in SH Westerly wind stress will have upon the AMOC. A high-resolution global ocean sea-ice model (MOM5) was used to induce a perturbation of +15% and -15% on the SH Westerly wind stress and the impact on major Atlantic currents and ocean characteristics (e.g. salinity, temperature and density) were quantified. It was found that a strengthening of the SH Westerlies resulted in greater transport of deep waters to the thermocline within the Southern Ocean. This increase in upwelling brought about an increase in salinity of Southern Ocean water masses. This, paired with the strengthening of key currents, resulted in more salt being transported to the north Atlantic deep water (NADW) formation sides leading to an increase in AMOC strength. An increase in AMOC strength could lead to extended periods of high pressure systems in the North Atlantic and North Europe that can cause droughts.

- Stuart-James Burney

Particle-Based Simulations Produce Spatiotemporal Patterns in Static and Growing Escherichia Coli Cells

Regular structures, or patterns, are ubiquitous in nature and form at all scales. In biological systems, pattern formation is not only responsible for intriguing visual phenomena, but it also underpins the workings of life itself. One of the key model systems for the study of protein pattern formation is the division of Escherichia Coli bacteria. The correct placement of the division site in the centre of the cell relies on the ability of a group of proteins, the Min system, to spontaneously form intracellular structures. This is an example of a pattern encoding global information, the location of the centre of the cell, even though the pattern forming proteins only exhibit local interactions. Many models of the dynamics of the Min proteins are formulated as deterministic partial differential equations. Whilst these models account for many features observed in vivo, they are unable to represent the inherent stochasticity that is present in the Min system due to the small number of proteins involved. The use of particle-based stochastic simulations provides a powerful tool to improve the existing deterministic models and investigate the notions of spatial-organisation and stochasticity in intracellular systems. In this thesis, improved models of the Min protein system are developed that consider the stochastic motion of the proteins as well as the growth of the cell over time. These models are formulated and solved via a stochastic simulation and a corresponding set of differential equations. A framework is presented to develop reaction-diffusion models on domains that are arbitrarily changing with time. The developed models are also employed to consider more geometrically complex examples arising in the division systems of Archaea.

- William Du

Modelling Lateral Diffusion on Cell Surfaces

Many biological activities occur on thin membrane layers which construct cell surfaces. The activities within these surfaces largely determine what is allowed in and out of cells and hence how those cells interact with the rest of the body. It is thus crucial to analyse the number and location of molecules embedded in these surfaces and how they laterally diffuse along it. An important delivery mechanism for transferring these molecules from within the cell to the surface is vesicle fusion, which can be experimentally observed under TIRF microscopes. However, there are two distinct mechanisms for delivery: Full fusion and Kiss-and-run, which are difficult to distinguish visually. Berk et al. demonstrate that mathematical modelling can simulate molecule movements through an analytical approach, with many limitations. A numerical approach removed much of the limitations and added distinct advantages, including the flexibility of solving for more complex geometries. More importantly, it allowed for a full-resolution simulation of molecule diffusion under TIRF. This further allowed for the determination of which delivery mechanism (Full fusion or Kiss-and-run) has been used and under what circumstances the difference becomes noticeable under regular TIRF microscopes.

- Yuan Yuan Wang

Application of the classical Moutard transformation to surfaces of constant negative Gaussian curvature

The classical Moutard equation may be regarded as a hyperbolic extension of the Schrödinger equation. In 1878, Moutard showed that this differential equation is form-invariant under a transformation which now bears his name. The Moutard transformation may be iterated algebraically, thereby generating sequences of solutions corresponding to different potentials. Remarkably, classical surfaces of constant negative Gaussian curvature are entirely encoded in a Moutard equation satisfied by the unit normal to these such surfaces, also known as pseudospherical surfaces. The aim of this thesis is to derive the action of the Moutard transformation on pseudospherical surfaces and present explicit examples generated in this manner. In fact, it will be shown that the Moutard transformation coincides with the classical Bäcklund transformation in the sense that, even though the former is applied algebraically, while the latter is defined geometrically, the action on pseudospherical surfaces turns out to be the same. The classical Bäcklund transformation hails all the way back to the 1800s, and its variants play a key role in integrable systems theory. The connection with integrable systems is briefly addressed by examining the action of the Moutard transformation on the Gauss equation for pseudospherical surfaces which happens to be an integrable hyperbolic partial differential equation, namely the sine-Gordon equation. As an application, the well-known kink solution of the sine-Gordon equation will be retrieved.

- Kabir Agrawal

Waves generated by moving bodies in two dimensions

Kelvin wakes are linear surface waves that form a distinctive V-shaped pattern behind moving boats. Kelvin wakes have been intensely studied since their first analysis by Lord Kelvin in 1886. Previous work uses linear wave theory, potential flow theory, and other simplifications to derive explicit solutions. This thesis simulates Kelvin wakes behind surface and subsurface vessels using spectral solvers and the volume penalization method, allowing us to move beyond assumptions underlying previous work. We analyse two dimensional (vertical slice) surface Kelvin wakes as well as wakes on internal interfaces in stratified fluids. We emphasise the importance of eddies in the turbulent and internal wave-making process, and show how these affect the dynamics of the waves. The observed speed of Kelvin waves closely matches predictions through potential flow theory along the transverse axis for submerged bodies. Further analysis of the structure of Kelvin wakes, through refining experimental parameters and considering three dimensional experiments, can provide further insight into the formation and evolution of Kelvin wakes. These will be useful in a variety of fields ranging from aerodynamical and acoustic engineering to maritime engineering and how to minimise wave making properties of these vessels.

- Josef Bisits

Passive tracer mixing along isopycnal surfaces in turbulent flows

Simulations of passive tracer experiments (in two dimensions - representing an isopycnal surface) are used to measure the mixing of passive tracers. This is done by relating the mixing of a single tracer to the stirring of an ensemble of tracers. Currently there are numerous methods and diagnostics to measure the diffusivity of passive tracers that have been shown to give broadly similar results over the course of an ensemble of experiments. These mainly focus on meridional diffusivity, here we also look at the full two dimensional diffusivity. We provide new diagnostics for the measurement of both meridional and full two dimensional diffusivity. Garrett (1983) showed that the area of a single tracer release (in two dimensions) asymptotes to the linear growth of the ensemble average area. The area of the ensemble is related to a horizontal diffusivity indicative of stirring by mesoscale eddies, so if the growth of the ensemble average area can be measured it can be related to a diffusivity. In this project we provide new diagnostics for measuring the area of tracer patches (and ensemble average areas) and relate this to a diffusivity. The experiments are simulations of the advection-diffusion of a concentration field (that is a passive tracer) in two dimensions using a two layer quasi-geostrophic flow. More information about the mixing of passive tracers along isopycnal surfaces will aid in carbon projections in the ocean (Gnanadesikan et al. 2015) and may provide more insight into long term stability of the climate (Sijp et al. 2006).

- Ming Hao

Laminar Flow in Channels with Porous Walls - with slip boundary conditions

One of the most famous models for laminar flow in porous channels is the Navier-Stokes equations, which describes the flow of a viscous, Newtonian fluid inside porous channels, applicable to many modern filtration systems. Our study provides new insights to the Navier-Stokes problem in two ways: considering a non-zero tangential velocity at the porous boundaries, and using fixed point theorem instead of finding a perturbation solution. Using Banach's fixed point theorem and a contractive mapping approach, we're able to prove, for a range of Reynold's numbers, a unique solution exists inside a thin strip around the homogeneous solution. This approach on the Navier-Stokes equations with slip boundary question is novel and improves our understanding for these problems, while creating new opportunities for combining pure mathematics techniques with traditionally applied mathematics problems.

- Lesley Gao

Pattern Formation in Fractional Order Dynamical Systems

Pattern formation modelled by reaction diffusion systems is the fundamental mechanism for modelling patterns in many areas, and especially in biology. Yet in recent decades, it has been shown that normal diffusion having a linear relationship between mean squared displacement and time is not appropriate for describing the diffusion processes in complex media. Instead, subdiffusion characterized by a sublinear power law between mean square displacement and time is a better representation of diffusion in these systems. This behaviour can be obtained from mathematical models of diffusion by introducing a fractional order time derivative. Reaction terms can then be incorporated as birth and death processes and the model equation for the evolution of species is referred to as a fractional reaction-subdiffusion model. There have been previous studies of pattern formation in these systems but they have been only been physically valid in cases where the reaction terms are strictly positive, or pure birth processes. In this thesis, we have investigated pattern formation in fractional reaction-subdiffusion models where both birth and death processes are included. The model systems that we have studied include a single species fractional Fisher equation and a two fractional species Gierer-Meinhardt equation.

- Bruce Cable

Pseudorandom Properties of the Subset Maps and Markoff Hash Function

In this thesis we investigate cryptographic properties of the subset sum and product generators, as well the Markoff hash function. Despite the formers potential significance for several cryptographic applications, very few theoretic results are known. Here we study experimentally their periodic properties and also present a theoretic result about the number of fixed points. Optimal parameters for the generator are determined through experimental results which ensure high entropy. The later has been proposed as a potential hash function, based on the difficulty of finding paths in the graph of markoff triples. We seek to analyse experimental evidence and decide whether further research is warranted.