



UNSW
SYDNEY

UNSW SCIENCE
School of Maths and Statistics

Course outline

MATH3161/MATH5165
Optimization

Term 1, 2022

Staff

Position	Name	Email	Room & Phone
Lecturer-in-charge	Professor Jeya Jeyakumar	v.jeyakumar@unsw.edu.au	RC-2073 (02) 9385 7046

Please refer to your Timetable on MyUNSW for your Lecture Tut, Lab enrolment days and times.

Timetable weblink:

MATH3161: <http://timetable.unsw.edu.au/2022/MATH3161.html>

MATH5165: <http://timetable.unsw.edu.au/2022/MATH5165.html>

The lectures will be common to both MATH3161 and MATH5165 students. Students in the graduate version (MATH5165) are expected to display much more independence, working through all tutorial problems in their own.

Administrative Contacts

Please visit the School of Mathematics and Statistics website for a range of information on School Policies, Forms and Help for Students.

For information on Courses, please go to “Current Students” and either Undergraduate and/or Postgraduate”, Course Homepage” for information on all course offerings,

The “Student Notice Board” can be located by going to the “Current Students” page; Notices are posted regularly for your information here. Please familiarise yourself with the information found in these locations. The School web page is: <https://www.maths.unsw.edu.au>

If you cannot find the answer to your queries on the web you are welcome to contact the Student Services Office directly.

By email Undergraduate ug.mathsstats@unsw.edu.au

Postgraduate pg.mathsstats@unsw.edu.au

By phone: 9385 7053 or 9385 7011

Should we need to contact you, we will use your official UNSW email address of in the first instance. **It is your responsibility to regularly check your university email account. Please state your student number in all emails.**

Course Information

Units of Credit: 6

Assumed knowledge / Pre-Requisite: 12 units of credit in Level 2 Mathematics courses including MATH2011 or MATH2111 or MATH2510, and MATH2501 or MATH2601, or both MATH2019(DN) and MATH2089, or both MATH2069(CR) and MATH2099.

Exclusions: MATH3181

We are aware some course exclusions on the Handbook may be different to the School website. We are in the process of updating this information. Meanwhile, students should be following the Handbook course information with the School website information as a supplement.

Course Aims

The concept of optimization, finding the “best” way to do something, arises across all branches of mathematics and in application areas ranging from data science and engineering to finance and medicine. The purpose of this course is to provide an introduction to the theory of multi-variable optimization and optimal control, and to provide students with the skills to formulate, solve and analyse solutions to certain multi-variable optimization problems and infinite dimensional optimal control problems.

This course has a major focus on nonlinear continuous optimization problems, as distinct from linear optimization problems and discrete optimization problems which are covered in MATH3171, Linear and Discrete Optimization Modelling, in Term 3.

Course Description

Overview: Optimization is an area of mathematics that directly deals with the problem of making the best possible choice from a set of numerous feasible choices. It seeks to understand how we achieve the best possible choice and how we can use this knowledge to improve management and technical decisions in science, engineering and commerce. Thinking in terms of choices is common in our cognitive culture and searching for the best possible choice is a basic human desire. There are countless examples where we make optimal (best) choices in our daily lives, such as finding the fastest way to work, searching for the shortest line at the supermarket checkout or finding the cheapest mobilephone plans. Thus, models of optimization arise everyday as management and technical decisions in many areas of human activity.

Problems of engineering design (such as the design of electronic circuits subject to a tolerancing and tuning provision), models of data science (such as the extraction of meaningful information from large databases and the classification of data), financial decision making and investment planning (such as the selection of optimal investment portfolios), and transportation management and so on arise in the form of a multi-variable optimization problem or an optimal control problem.

Teaching Strategies and Rationale Underpinning the Course

New concepts and skills are first introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in tutorials and assessments.

We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence, this course is structured with a strong emphasis on problem-solving tasks in lectures, tutorials and in assessment tasks, and students are expected to devote the

majority of their class and study time to the solving of such tasks.

To ensure effective learning, students should participate in class as outlined below.

We believe that effective learning is achieved when students attend all classes, have prepared effectively for classes by reading through previous lecture notes, in the case of lectures, and, in the case of tutorials, by having made a serious attempt at doing for themselves the tutorial problems prior to the tutorials.

Furthermore, lectures should be viewed by the student as an opportunity to learn, rather than just copy down lecture notes.

Effective learning is achieved when students have a genuine interest in the subject and make a serious effort to master the basic material.

The art of logically setting out mathematics is best learned by watching an expert and paying particular attention to detail. This skill is best learned by regularly attending classes and watching supplementary course videos.

Assessment and Deadlines

Assessment	Week	Weighting %	Course Learning Outcome (CLO)
On-line Class Test 1; 40 minutes	5	15%	1, 2, 3
On-line Class Test 2; 40 minutes	9	20%	1,2,3,4
Assignment	10	5%	1,2,3,4, 5
Final Exam		60%	All

Starred Materials: Problem sheets, class tests and the final exam may have starred questions indicating harder material. Grades of Pass and Credit can be gained by satisfactory performance on unstarred questions. Grades of Distinction and High Distinction will require satisfactory performance on all questions.

Students in the graduate version (MATH5165) are expected to show satisfactory performance on starred questions.

Assignment: It is planned to have an assignment on optimization methods to solve practical problems. The assignment may involve modelling practical problems and writing a short report. The *Matlab* software package may also be used for implementing numerical optimization methods to solve practical optimization problems. You are not required to know *Matlab* before this course. On-line help on *Matlab* will be available. It is not assumed that you have done computing subjects.

Students in the graduate version (MATH5165) are expected to complete additional work in the assignment.

Late Submission of Assessment Tasks

No late submissions will be accepted. (Where "late" in this context means after any extensions granted for Special Consideration or Equitable Learning Provisions.)

Course Learning Outcomes (CLO)

Students taking this course will develop an appreciation of the basic problems of optimization and skills to solve optimization problems.

By the end of the course, students should be able to:

1. State definitions and theorems in the syllabus and apply them to specific examples.
2. Apply the concepts and techniques of the syllabus to solve appropriate mathematical problems.
3. Solve optimization problems via analytical, numerical and computational methods.
4. Recognize and create valid optimization models and apply correct mathematical techniques.
5. Use technology as an aid to solve optimization models and communicate mathematical outcomes.

Course Schedule

The course will include material taken from some of the following topics. This should only serve as a guide as it is not an extensive list of the material to be covered and the timings are approximate. The course content is ultimately defined by the material covered in lectures and provided in Moodle.

Weeks	Topic
1 Lecture 1-4	Optimization - What is it? Modelling; standard form formulations, norms, existence, relaxation, gradients and Hessians; positive definite matrices.
2 Lecture 5-8	Convexity of Sets and Functions: Convex sets, extreme points, convex combinations, convex functions, epigraphs, extrema of convex functions
3 Lecture 9-12	Optimization: unconstrained & Equality constraints: First order optimality principles; Second-order optimality principles; necessary conditions; sufficient conditions; convexity and global optimality conditions, Equality constraints, regularity conditions, method of Lagrange multipliers; first-and second-order optimality conditions
4 Lecture 13-16	Optimization: inequality constraints, global optimality and duality: KKT conditions; convex optimization; necessary and sufficient global optimality conditions; duality; right-hand side perturbations
5 Lecture 17-20	Numerical Methods: Rates of convergence, iterative methods, descent methods, line search methods; steepest descent methods.
6 TERM BREAK	
7 Lecture 21-24	Newton and conjugate gradient methods: Basic Newton's methods, conjugate gradient methods
8 Lecture 25-28	Penalty methods & introduction to optimal control: Penalty functions, penalty function methods for constrained optimization, optimal control models.
9 Lecture 29-32	Optimal control problems & PMP: systems of differential equations; Pontryagin Maximum Principle (PMP)
10 Lecture 33-36	Applications of PMP: Autonomous control problems with fixed targets, free time problems; Extension of PMP to problems with general targets; non-autonomous problems.

Textbooks and Additional Resources and Support

There is **NO textbook** which covers all aspects of this course.

Tutorial Exercises and Videos

Problem sheets for tutorials will be provided via UNSW Moodle. These problems are for you to do to enhance mastery of the course.

SOME of the problems will be done in tutorials, but you will learn a lot more if you try to do them before the tutorial. Tutorial videos will demonstrate how hard tutorial problems are solved.

Lecture Notes and Mini-Lecture Videos

A set of skeleton notes and summary sheets containing only definitions, theorems and proofs will be provided for SOME components of the course on UNSW Moodle.

Mini-lecture videos will cover some background materials for the course and give proofs of key optimization theorems.

On-line Quizzes

On-line quizzes will give you multiple opportunities to practice the key concepts and techniques, learned in the lectures and tutorials, and provide instant feedback on answers.

Sample test videos

Sample test videos will cover some past class test questions.

Calculators

You may bring your own UNSW approved Scientific Calculator to the class tests and the final exam.

Moodle

Log in to Moodle to find announcements, general information, notes, lecture slide, classroom tutorial and assessments etc.

<https://moodle.telt.unsw.edu.au>

School and UNSW Policies

The School of Mathematics and Statistics has adopted a number of policies relating to enrolment, attendance, assessment, plagiarism, cheating, special consideration etc. These are in addition to the Policies of The University of New South Wales. Individual courses may also adopt other policies in addition to or replacing some of the School ones. These will be clearly notified in the Course Initial Handout and on the Course Home Pages on the Maths Stats web site.

Students in courses run by the School of Mathematics and Statistics should be aware of the School and Course policies by reading the appropriate pages on the Maths Stats web site starting at:

<https://www.maths.unsw.edu.au/currentstudents/assessment-policies>

The School of Mathematics and Statistics will assume that all its students have read and understood the School policies on the above pages and any individual course policies on the Course Initial Handout and

Course Home Page. Lack of knowledge about a policy will not be an excuse for failing to follow the procedure in it.

Academic Integrity and Plagiarism

UNSW has an ongoing commitment to fostering a culture of learning informed by academic integrity. All UNSW staff and students have a responsibility to adhere to this principle of academic integrity. Plagiarism undermines academic integrity and is not tolerated at UNSW. *Plagiarism at UNSW is defined as using the words or ideas of others and passing them off as your own.*

The **UNSW Student Code** provides a framework for the standard of conduct expected of UNSW students with respect to their academic integrity and behaviour. It outlines the primary obligations of students and directs staff and students to the Code and related procedures.

In addition, it is important that students understand that it is not permissible to buy essay/writing services from third parties as the use of such services constitutes plagiarism because it involves using the words or ideas of others and passing them off as your own. Nor is it permissible to sell copies of lecture or tutorial notes as students do not own the rights to this intellectual property.

If a student breaches the Student Code with respect to academic integrity, the University may take disciplinary action under the **Student Misconduct Procedure**.

The UNSW Student Code and the Student Misconduct Procedure can be found at: <https://student.unsw.edu.au/plagiarism>

An online Module “[Working with Academic Integrity](https://student.unsw.edu.au/aim)” (<https://student.unsw.edu.au/aim>) is a six-lesson interactive self-paced Moodle module exploring and explaining all of these terms and placing them into your learning context. It will be the best one-hour investment you’ve ever made.

Plagiarism

Plagiarism is presenting another person's work or ideas as your own. Plagiarism is a serious breach of ethics at UNSW and is not taken lightly. So how do you avoid it? A one-minute video for an overview of how you can avoid plagiarism can be found <https://student.unsw.edu.au/plagiarism>.

Additional Support

ELISE (Enabling Library and Information Skills for Everyone)

ELISE is designed to introduce new students to studying at UNSW.

Completing the ELISE tutorial and quiz will enable you to:

- analyse topics, plan responses and organise research for academic writing and other assessment tasks
- effectively and efficiently find appropriate information sources and evaluate relevance to your needs
- use and manage information effectively to accomplish a specific purpose
- better manage your time
- understand your rights and responsibilities as a student at UNSW

- be aware of plagiarism, copyright, UNSW Student Code of Conduct and Acceptable Use of UNSW ICT Resources Policy
- be aware of the standards of behaviour expected of everyone in the UNSW community
- locate services and information about UNSW and UNSW Library

Some of these areas will be familiar to you, others will be new. Gaining a solid understanding of all the related aspects of ELISE will help you make the most of your studies at UNSW.

The *ELISE* training webpages:

<https://subjectguides.library.unsw.edu.au/elise/aboutelise>

Equitable Learning Services (ELS)

If you suffer from a chronic or ongoing illness that has, or is likely to, put you at a serious disadvantage, then you should contact the Equitable Learning Services (previously known as SEADU) who provide confidential support and advice.

They assist students:

- living with disabilities
- with long- or short-term health concerns and/or mental health issues
- who are primary carers
- from low SES backgrounds
- of diverse genders, sexes and sexualities
- from refugee and refugee-like backgrounds
- from rural and remote backgrounds
- who are the first in their family to undertake a bachelor-level degree.

Their web site is: <https://student.unsw.edu.au/els/services>

Equitable Learning Services (ELS) may determine that your condition requires special arrangements for assessment tasks. Once the School has been notified of these, we will make every effort to meet the arrangements specified by ELS.

Additionally, if you have suffered significant misadventure that affects your ability to complete the course, please contact your Lecturer-in-charge in the first instance.

Academic Skills Support and the Learning Centre

The Learning Centre offers academic support programs to all students at UNSW Australia. We assist students to develop approaches to learning that will enable them to succeed in their academic study. For further information on these programs please go to:

<http://www.lc.unsw.edu.au/services-programs>

Applications for Special Consideration for Missed Assessment

Please adhere to the Special Consideration Policy and Procedures provided on the web page below when applying for special consideration.

<https://student.unsw.edu.au/special-consideration>

Please note that the application is not considered by the Course Authority, it is considered by a centralised team of staff at the Nucleus Student Hub.

The School will contact you (via student email account) after special consideration has been granted to reschedule your missed assessment, for a *lab test or paper-based test* only.

For applications for special consideration for *assignment extensions*, please note that the new submission date and/or outcome will be communicated through the special consideration web site only, no communication will be received from the School.

For Dates on Final Term Exams and Supplementary Exams please check the “Key Dates for Exams” ahead of time to avoid booking holidays or work obligations.

<https://student.unsw.edu.au/exam-dates>

If you believe your application for Special Consideration has not been processed, you should email specialconsideration@unsw.edu.au immediately for advice.

Course Evaluation and Development (MyExperience)

Student feedback is very important to continual course improvement. This is demonstrated within the School of Mathematics and Statistics by the implementation of the UNSW online student survey *myExperience*, which allows students to evaluate their learning experiences in an anonymous way. *myExperience* survey reports are produced for each survey. They are released to staff after all student assessment results are finalised and released to students. Course convenor will use the feedback to make ongoing improvements to the course.

Reference Books

The general references on optimization are listed below. The standard of the references is somewhat higher than is required in MATH3161/MATH5165.

Optimization References: General references on multi-variable optimization include [1, 2, 5, 11] and on optimal control include [9, 12]

Linear Algebra and Differential Equations: Solving multi-variable optimization problems requires techniques from linear algebra, whereas solving optimal control problems requires solution methods of differential equations. An elementary treatment of linear algebra can be found in Strang [14], while a reference for differential equations is Zill [15].

Mathematical Software: Solving practical problems typically requires a computer software package like MATLAB [10] (see Pratap [13] for an introduction).

References

1. F.J. Aragon-Artacho, M.A. Goberna, M.A. López, M.M.L. Rodríguez, Nonlinear Optimization, Springer, 2019.
2. A. Beck, Introduction to Nonlinear Optimization – Theory, Algorithms and Applications with MATLAB, MOS-SIAM Series on Optimization. SIAM, 2014.
3. D. P. Bertsekas Nonlinear programming: Second edition, Athena Scientific, Belmont, MA, 1999.
4. J. E. Dennis and R. B. Schnabel, Numerical Methods for Unconstrained Optimization and Nonlinear Equations, SIAM Publications, Classics in Applied Mathematics, 1996.
5. R. Fletcher, Practical Methods of Optimization, 2nd Edition, John Wiley, 2000.
6. P. E. Gill, W. Murray, and M. H. Wright, Practical Optimization, Academic Press, New York and London, 1981.
7. G. H. Golub and C. F. Van Loan, Matrix Computations, John Hopkins University Press, Baltimore and London, third ed., 1996.
8. J. B. Hiriart-Urruty and C. Lemarechal, Convex Analysis and Minimization Algorithms, Springer-Verlag, Berlin, 1993
9. L. M. Hocking, Optimal Control: An Introduction to the Theory with Applications, Oxford University Press, Oxford, 1991.
10. MathWorks, MATLAB & Simulink Student Version R2012A, Englewood Cliffs, 2012.
11. J. Nocedal and S. J. Wright, Numerical optimization, Springer, (2nd edition) 2006.
12. E. R. Pinch, Optimal control and the calculus of variations, Oxford University Press, Oxford, 1995.
13. R. Pratap, Getting started with MATLAB: A Quick Introduction for Scientists and Engineers, Oxford University Press, 2009.
14. G. Strang, Linear Algebra and its Applications, Harcourt Brace Jovanovich, San Diego, 3 ed., 1988.
15. D. G. Zill, Differential equations with boundary-value problems, Second Edition, PWS-Kent Publishing company, Boston, 1989.