



UNSW
SYDNEY

UNSW SCIENCE
School of Maths and Statistics

Course outline

MATH5825
Measure, Integration and Probability

Term 3, 2021

MATH5825 – Course Outline

Information about the course

Course Authority and Lecturer: Dr Dmitriy Zanin, email d.zanin@unsw.edu.au

Consultation: Arrange an appointment via email.

Credit: This course counts for 6 Units of Credit (6UOC).

Prerequisites: MATH3611 (or an equivalent background in abstract analysis) is assumed knowledge for this course.

Timetable: The meetings for this course are as follows:

| | | |
|-----------|-------------|--------|
| Wednesday | 14.00-16.00 | online |
| Friday | 10.00-12.00 | online |

Tutorials: From time to time we'll use one of the slots for a tutorial (normally 1 hour each week).

Course aims

Measure theory provides a foundation for many branches of mathematics such as functional analysis, harmonic analysis, ergodic theory, theory of partial differential equations and probability theory. It is a central, extremely useful part of modern analysis, and many further interesting generalizations of measure theory have been developed. It is also subtle, with surprising, sometimes counter-intuitive, results. The aim of this course is to learn the basic elements of Measure Theory, and particular focus will be given to applications in probability theory and statistics.

Tentative Course Schedule

The course will cover the following topics.

1. Problems of the Riemann integral. Lebesgue's "problem of measure" in \mathbb{R}^d .
2. Abstract measure theory - σ -algebras, measurable sets, measures, outer measures, Lebesgue measure and its properties, completion of measures.
3. Measurable functions, approximation by simple functions.
4. Lebesgue integral, Monotone Convergence Theorem, Dominated Convergence Theorem, coincidence of Lebesgue and Riemann integral for Riemann integrable functions.
5. Probabilistic language. Random variables, expectation.
6. L^p spaces.
7. Signed measures, Hahn decomposition theorem, Jordan decompositions, absolute continuity of measures, Lebesgue decomposition, Radon–Nikodym Theorem, Radon–Nikodym derivatives, chain rule
8. Weak convergence of measures. Convergence in measure.
9. Conditional expectations. Martingales. Martingale Convergence Theorems

Time permitting, we might also touch on:

- Product measures, Tonelli–Fubini theorem.
- Measures on infinite dimensional spaces, Gaussian measures.

Assessment

Assessment in this course will consist of:

| Assessment Task | Weight | Date |
|-----------------|--------|---------|
| Assignment 1 | 10% | Week 3 |
| Assignment 2 | 20% | Week 6 |
| Assignment 3 | 20% | Week 10 |
| Final Exam | 50% | |
| Total: | 100% | |

The task for Assignments 1,2 and 3 will be posted on Moodle on Wednesday of the respective week (so that you can ask questions about the statement/format/marking of the task during Wednesday lecture). The deadline is 14.00-16.00 on Wednesday next week.

Assignments: To test your ability to solve nontrivial problems using the material in the course. Draconian late penalties for assignments will apply. Students seeking relief from such penalties should discuss their situation with the lecturer at the earliest possibility.

Course material

Notes and problem sheets will be available on Moodle.

There is no set textbook for this course, but the following books will be helpful:

- G.B. Folland, Real Analysis, Wiley 1984.
- W. Rudin, Real and complex analysis. McGraw-Hill, 1987.
- P. Billingsley, Probability and Measure, P519.1/492
- P.R. Halmos, Measure theory, P517.52/24
- W. Rudin, Functional analysis. McGraw-Hill, 1991.
- H.L. Royden, Real Analysis, McMillan, 1978.
- J.L. Doob, Measure theory, P517.52/171
- A.N. Kolmogorov and S.V. Fomin, Introductory Real Analysis, Dover, 1975. (Cheap!)

Course Evaluation and Development

The School of Mathematics and Statistics evaluates each course each time it is run. We carefully consider the student responses and their implications for course development. It is common practice to discuss informally with students how the course and their mastery of it are progressing.

Student Learning Outcomes

Students taking this course will develop an appreciation of the basic concepts of measure theory. These methods will be useful for further study in a range of other fields, e.g. Stochastic calculus, Quantum Theory and Harmonic analysis.

Relation to graduate attributes: The above outcomes are related to the development of the Science Faculty Graduate Attributes, in particular: 1. Research, inquiry and analytical thinking abilities, 4. Communication, 6. Information literacy

Teaching strategies underpinning the course

New ideas and skills are introduced and demonstrated in lectures, then students develop these skills by applying them to specific tasks in problem sheets and assessments.

Rationale for learning and teaching strategies

We believe that effective learning is best supported by a climate of inquiry, in which students are actively engaged in the learning process. Hence this course is structured with a strong emphasis on problem-solving tasks in assessment tasks, and students are expected to devote the majority of their class and study time to the solving of such tasks.

Rationale for Assignments: Assignments will give an opportunity for students to try their hand at more difficult problems requiring more than one line of argument and also introduce them to aspects of the subject which are not explicitly covered in lectures.

Rationale for Examinations: The final examination will assess student mastery of the material covered in the lectures.

Assessment criteria: The main criteria for marking all assessment tasks will be clear and logical presentation of correct solutions.

Knowledge and abilities assessed: All assessment tasks will assess the learning outcomes outlined above.

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Administrative matters

School Rules and Regulations: Fuller details of the general rules regarding attendance, release of marks, special consideration etc. are available at <http://www.maths.unsw.edu.au/currentstudents/assessment-policies>. For UNSW policies, procedures and guidelines see <https://student.unsw.edu.au/policy>

Equity and diversity issues

Students with any special needs and requests should either see the lecturer as soon as possible, or ensure that SEADU sends appropriate documentation.

Similarly, any student whose study is being adversely affected by external issues should see the lecturer and/or get an appropriate counsellor/doctor/... to contact the lecturer.

Plagiarism and academic honesty

Plagiarism is the presentation of the thoughts or work of another as one's own.

Issues you must be aware of regarding plagiarism and the university's policies on academic honesty and plagiarism can be found at

<http://student.unsw.edu.au/plagiarism>

You should also be aware that plagiarism rules vary significantly from discipline to discipline. Some issues that are mathematics specific are discussed below.

Plagiarism in mathematics

The plagiarism rules **do not** preclude you from consulting references and discussing questions with other people. Indeed you can often learn a great deal from such discussions. It just means that you need to be clear and honest about where the ideas came from. This can usually be covered by a sentence such as 'This proof is based on the proof of Theorem 3.2.1 in ...' or 'The clever idea to consider random subsets of Γ was provided by Kylie'.

Note in particular that:

- **You will not lose marks because you found the answer in a book (great research!) or were helped by a friend.**
- **You will lose marks if you don't say where you found a solution, or who helped you, or if you copy something without understanding what you have written.**

You don't need to reference definitions, theorems etc that you would expect everyone in the course to know (eg the Mean Value Theorem, or the definition of a metric space, or something from the lectures). On the other hand, if your proof depends on a slightly more unusual theorem (eg Jensen's formula), then you should (a) check that that theorem doesn't depend on the thing that you have been asked to prove, and (b) give a reference to where you found the result. Your solutions should be written so that a fellow student could read and follow what you have done — keep this in mind as you are writing up your work.

Obviously straight copying of someone else's assignment is both unethical and easily detected. You should never provide your written solutions to another student; inevitably their work will look just like yours and you'll both be penalized. If you work on a problem with a friend, make sure that you each write it up quite separately (and mention that you discussed the problem with your classmate).

There has been an increasing tendency for many students to resort to consulting Dr Google as a *first step* in trying to solve a problem. This should be avoided, not because we discourage researching information, but because it will stop you developing the ability to attack hard problems whose solution is not to be found on the internet. Read the notes, work out what is being asked and think about it for a while. Discuss the problem with your classmates and your lecturer.

Similarly, you should never copy word-for-word something that find in a text/web-page/... In mathematics there is a limit to how much one can rearrange a proof to make it look like yours, so we allow greater degrees of similarity than would be acceptable in many other disciplines. However it is almost never the case that you find a proof that has the same sort of notation that we are using, exactly the same definitions and theorems etc. You always need to adapt these things to the context of the course that you are doing.

It is **very** easy to tell when someone has copied something and not understood it.

Serious cases of plagiarism will attract severe penalties.

Grading criteria

Roughly speaking, here is what I expect for the various grades:

| Grade | Standard |
|-------|--|
| PS | Knowledge of the main concepts, definitions and theorems. Ability to produce a simple proof using these, given time. |
| CR | Shows understanding of the main concepts, definitions and theorems. Knows some examples. Can make a decent attempt at more complex proofs in assignments and can manage simple proofs under exam conditions. |
| DN | No major misunderstandings of the concepts and a good knowledge of main examples. Only misses more subtle points in most proofs in assignments and can have a decent go at harder proofs in the exam. |
| HD | Excellent understanding of the concepts and examples in the course. Proofs mainly watertight, especially in the assignments. For a top HD a level of mathematical flair and sophistication is expected. |

Depending on the ability of the class, the exam will be set to best allow me to test the above things and to differentiate between students. The performance bands in the exam will be mapped to the appropriate standard mark ranges (eg PS 50—64, HD 85—100) before being added to the assignment marks to produce a preliminary mark. Closer to the end of session I will give the class some idea of what sort of expectations I have for raw marks in the final exam.

We may do a small degree of final scaling to the preliminary mark to avoid grade boundaries ending up in unfortunate places and to ensure consistency between courses.