MATHEMATICS ENRICHMENT CLUB.¹
Problem Sheet 13, August 20, 2012

1. Laurie sold two cars for $25 000 each. One he sold at a 20% profit and the other at a 20% loss. How much did he gain or lose?

2. A pizza has radius $z$, and height $a$. What is interesting about it’s volume?²

3. If a triangle $ABC$ has sides of length $a, b, c$ such that $a^2 + b^2 = c^2$, prove that it must be a right-angled triangle.

4. Without using a calculator, which is larger $31^{24}$ or $257^{15}$.

5. Let $S_n = 2n(2n - 1)(2n - 2)...(n + 1)$. For example, $S_3 = 6 \times 5 \times 4 = 120$.
   (a) What is the power of 2 in the prime factorisatoin of $S_n$ for $n = 2, 3, 4...$?
   (b) Make a conjecture based on(i) and prove it.

6. Without using a calculator, show that
   \[
   \sqrt[3]{5\sqrt{13} + 18} - \sqrt[3]{5\sqrt{13} - 18} = 3.
   \]
   (Hint: Let $x = a - b$ and cube.)

7. Let $ABC$ be a triangle and $D, E$ points on $AB, BC$ respectively, and $S$ be the intersection of $AE$ and $CD$. If $AD = DB$ and $BE : EC = 2 : 1$, find the ratios $CS : SD$ and $AS : SE$.

8. (a) Let $P$ be an interior point in an equilateral triangle $ABC$. Prove that we can always form a triangle with sides of length $AP, BP, CP$. (That is, we have to show that the sum of any two of these lengths is larger than the remaining one.)
   (b) Give an example of a triangle and point inside it for which the above result is not true.

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.
²This question thanks to Mike Hirschhorn