

## MATHEMATICS ENRICHMENT CLUB.<sup>1</sup>

### Problem Sheet 14, August 27, 2012

1. In how many ways can three boys and three girls stand in a line so that no two boys are next to each other and no two girls are next to each other?
2. Draw a regular 5-pointed star. Find the sum of the angles at the 5 points of the star. What happens in an  $n$ -pointed star?
3. Three vertices of a cube in space are  $A(1, 3, 5)$ ,  $B(3, 3, 5)$  and  $C(3, 5, 7)$ . Find the coordinates of the centre of the cube and draw the cube showing the coordinates of each of its vertices.
4. Find the difference between the sum of the first 80 positive even integers and the first 80 positive odd integers.
5. How many pairs of integers  $x, y$  satisfy  $|x| + |y| = 0, 1, 2, 3..?$  Hence find the number of pairs of integers such that  $|x| + |y| < 2000$ .
6. (a) Show that if the diagonals of a parallelogram are perpendicular, then the figure is a rhombus.  
(b) Show that if the diagonals of a parallelogram are equal then the figure is a rectangle.
7. Describe how to construct a triangle  $ABC$  if we are given the lengths of the sides  $AB$  and  $AC$  and the length of the median  $AM$  drawn from  $A$ .

### Senior Questions

1. Take any (fixed) positive integer  $m$  and let  $P_n = (m + 1)(m + 2) \dots (m + n)$ . Prove that  $n!$  is always a factor of  $P_n$ . (For example,  $6! = 720$  is a factor of  $8 \cdot 9 \cdot 10 \cdot 11 \cdot 12 \cdot 13 = 1,235,520$ .)
2. Differentiate  $\log(x + \sqrt{x^2 - a^2})$  with respect to  $x$ .
3. Find the stationary point(s) of  $f(x) = x^{2x}$ , and prove that the graph of  $f$  is always concave up.

---

<sup>1</sup>Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.