## MATHEMATICS ENRICHMENT CLUB.[1]
## Problem Sheet 2, May 7, 2012

1. Find all pairs of primes $(p, q)$ such that $p$ divides $q^2 - q$ and $q$ divides $p^2 + p$.

2. If $x$ is a number between 4 and 8 and $y$ is a number between 20 and 40, what are the smallest and largest possible values of $\frac{y}{x}$?

3. Write the quartic $x^4 + 4$ as the product of two quadratics. What about $x^4 + 1$?

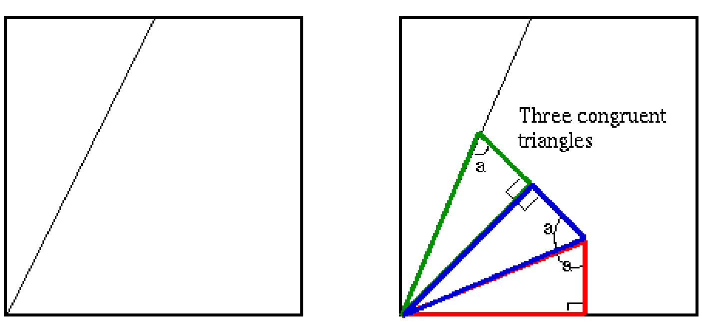
4. Find all positive integers $x, y, z$ such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{8}$.
(Hint: Suppose $x \leq y \leq z$ and hence find the possible values of $x$.)

5. Suppose that a set $S$ contains the numbers 1,2,3,4 and that the sum of any four different numbers in $S$ is also a number in $S$. Show that $S$ contains every positive integer greater than 21.

6. Let $ABC$ be a triangle whose incircle has radius $r$. Let $s$ equal to half the perimeter of the triangle. Show that the area of the triangle is $rs$.

7. Let $S$ be the centroid of the triangle $ABC$ and let $A_1, B_1, C_1$ be the midpoints of the sides, (with $A_1$ opposite $A$ and so on). If $AA_1$ meets $B_1C_1$ at $G$, find the ratio of the areas of $B_1GS$ to the area of $ABC$.

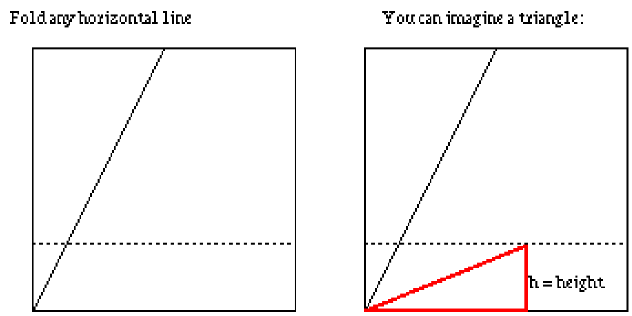
8. Tricsecting the angle (taken from www.math.lsu.edu/ verrill/origami/trisect/)
Suppose we could put three congruent triangles in the picture as shown:



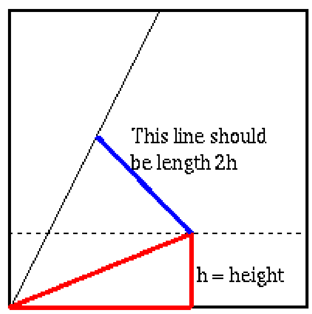

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

These triangles trisect the angle. So we need to know how to get them there.

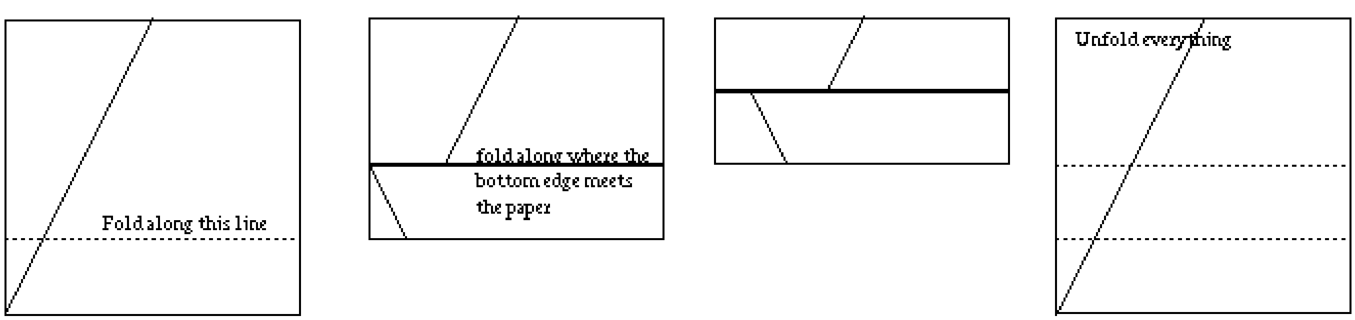
Choose some height for the lower triangle, any height, and crease a horizontal line at this height; ie, just crease any horizontal line you want:



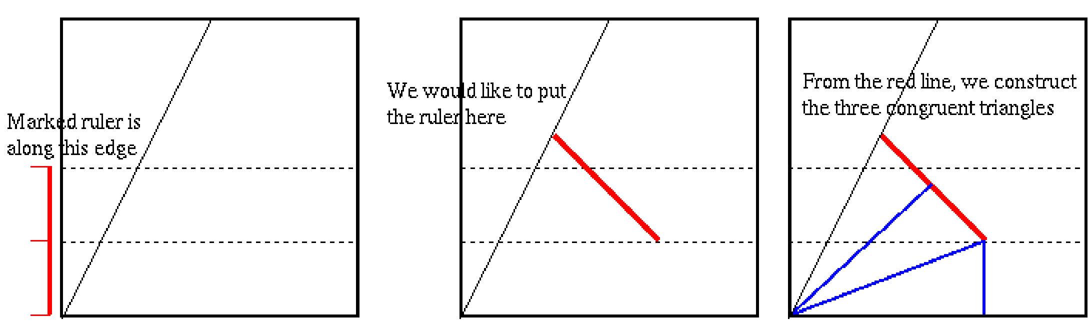

We need to get the blue line of the following picture somehow:



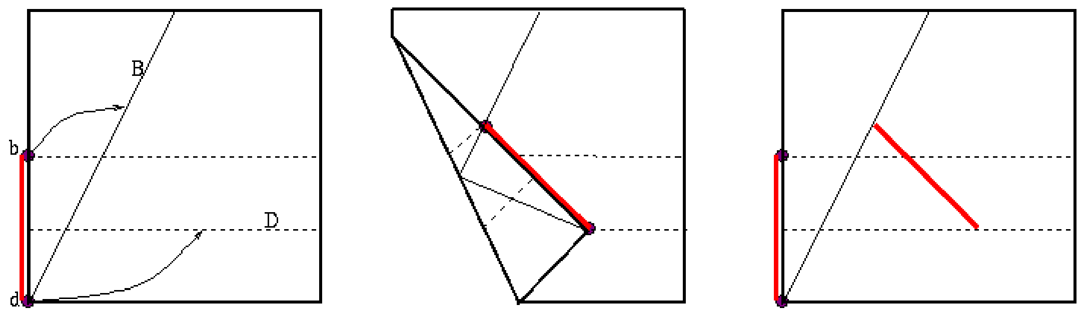

We can make a kind of ”marked ruler” in the side of the paper, by folding over the paper again:




Now this ”marked ruler” is used to find the bold line we needed:



To do this, fold the paper so point b touches line B, and point d touches line D:




## Senior Questions.

1. If $x$ is positive, how large must $x$ be so that $\sqrt{x^2 + x} - x$ differs from $\frac{1}{2}$ by less than 0.02?

2. Find the largest integer that exactly divides $11^{k+2} + 12^{2k+1}$ for all positive integers $k$.

3. Solve the equation $\cot^{-1} x - \cot^{-1}(x + 2) = \frac{\pi}{12}$.