UNSW
THE UNIVERSITY OF NEW SOUTH WALES

Never Stand Still | Faculty of Science | School of Mathematics and Statistics

MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 5, May 28, 2012

1. Two classes of 20 and 30 students average 66% and 56% respectively on an examination. What is the average for all the students on the exam?

2. A mathematics test has 5 questions on each of which people can score 0,1,2 or 3 marks. How many ways can a student receive a total of 12 marks for the test?

3. Mark the hours on a clockface with centre O with the letters $A_1, A_2, ..., A_{12}$.

   (a) Find all the angles XYO, where X and Y are any hours.

   (b) What is the ratio of the areas of the quadrilaterals $A_{12}A_2A_6A_8$ and $A_{12}A_3A_6A_9$?

4. Find infinitely many integers x such that

$$\sqrt[3]{x + \sqrt{x^2+1}} + \sqrt[3]{x - \sqrt{x^2+1}}$$

   is an integer.

5. (a) Prove that $a + b \geq 2\sqrt{ab}$ for any positive real numbers a, b.

   (b) Deduce that for x, y, z positive, $(x + y)(x + z)(y + z) \geq 8xyz$.

6. In the triangle ABC, it is given that $\angle ABC = 140°$. Let D be a point on AC and E a point on AB such that the three triangles AED, EDB and DBC are all isosceles, with their vertices at E, D and B respectively. Find all the angles of the triangle ABC.

7. Let $ABCD$ be a trapezium and with $AB||CD$. Let M, N be the midpoints of AD and BC respectively. Show that $MN = \frac{1}{2}(AB + CD)$.

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

Senior Questions.

1. Let $f(x) = \left(1 + \dfrac{1}{x}\right)^x$.

   (a) Prove that $\dfrac{f'(x)}{f(x)} = \log\left(1 + \dfrac{1}{x}\right) - \dfrac{1}{1+x}$.

   (b) By considering the area under the curve $y = \frac{1}{t}$ for t from 1 to $1 + \frac{1}{x}$, show that $\log\left(1 + \dfrac{1}{x}\right) > \dfrac{1}{1+x}$ and deduce that $f(x)$ is increasing.

2. Suppose $a > b > 0$. Find $\lim_{n\to\infty} (a^n + b^n)^{\frac{1}{n}}$.

3. By considering $\cos(A + B) + \sin(A - B) = 0$ find the general solution (for θ) of $\cos n\theta + \sin m\theta = 0$.