MATHEMATICS ENRICHMENT CLUB.\textsuperscript{1}
Problem Sheet 8, June 20, 2012

1. The last digit of $1997^{1997}$ is
   \[(a) \ 1 \quad (b) \ 3 \quad (c) \ 5 \quad (d) \ 7 \quad (e) \ 9.\]

2. The number of positive integers less than 1000 which are divisible neither by 3 nor 4 is
   \[(a) \ 582 \quad (b) \ 499 \quad (c) \ 500 \quad (d) \ 7501 \quad (e) \ None \ of \ these.\]

3. Draw a right triangle $ABC$ with right-angle at $C$ and the sides marked $a, b, c$.\textsuperscript{2}
   \[\text{(a) Draw the enlargement } A'B'C' \text{ of } ABC \text{ by a factor of } a.\]
   \[\text{(b) On the same diagram draw the enlargement } A''B''C'' \text{ of } ABC \text{ by a factor of } b, \]
   \[\text{lining up } B'C' \text{ with } A''C'', \text{ so that } A'' = B', \ C'' = C', \text{ and } A', C' \text{ and } B'' \text{ are} \]
   \[\text{collinear, and thus form a new triangle } A'A''B''.\]
   \[\text{(c) Explain why the angle } A'A''B'' \text{ is a right angle.}\]
   \[\text{(d) What theorem have you just proven and why?}\]

4. Find all positive integer solutions to
   \[\frac{1}{x} + \frac{1}{y} = \frac{1}{6}.\]

5. (a) Find the greatest common divisor of $2^{50} + 1$ and $2^{20} + 1$.\textsuperscript{1}
   \[\text{(b) Explain why the greatest common divisor of } 2^m + 1 \text{ and } 2^n + 1 \text{ is at least 3 if } m \]
   \[\text{and } n \text{ are both odd.}\]

6. (a) Prove that the angle in a semicircle is right-angle.
   \[\text{(b) Show that if two chords of a circle mutually bisect each other, then they are both} \]
   \[\text{diameters.}\]
   \[\text{(c) Complete the following statement: If a parallelogram is inscribed in a circle then} \]
   \[\text{.........}\]

7. (a) Given an equilateral triangle, what is the smallest number of non-overlapping \textsuperscript{1}
   \[\text{equilateral triangles into which it can be dissected?}\]
   \[\text{(b) Describe how to dissect it into seven equilateral triangles.}\]

\textsuperscript{1} Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.
\textsuperscript{2} Thanks to Mike Hirschhorn for this problem
Senior Questions

1. Evaluate

\[ \lim_{x \to \infty} \frac{3x^2 + \sin(2x^2)}{x^2} \]

2. Evaluate

\[ \lim_{x \to 0} \frac{3x^2 + \sin(2x^2)}{x^2} \]