MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 9, July 26, 2012

1. How many ways can we change $1.00 into coins of 5, 10, 20 and/or 50 cents?
   
   a. 46    b. 47    c. 48    d. 49    e. 50.

2. (a) What number is one less than 11010000 in base 3?
   
   (b) What is the decimal representation of $220200_3$?

3. If we expand $(2 + x)^{18}$ as a polynomial we obtain
   
   $$(2 + x)^{18} = a_0 + a_1x + a_2x^2 + \ldots + a_{18}x^{18},$$
   
   where $a_0, a_1, \ldots, a_{18}$ are integers.

   (Without using the binomial theorem), find $a_0, a_1, a_{18}$ and $a_0 + a_1 + \ldots + a_{18}$.

4. The hypotenuse of a right-angled triangle is 15 cm and the radius of the inscribed circle is 2 cm. Find the perimeter of the triangle.

5. Show that if $A$ is any subset containing $n + 1$ integers from the set \{1, 2, 3, ..., $2n$\}, then $A$ contains (at least) 2 integers $a$ and $b$ such that $a$ is a factor of $b$.

6. (a) How many diagonals are there in a regular pentagon $ABCDE$? Show that each diagonal is parallel to one of the sides.

   (b) How many diagonals are there in a regular hexagon. Is there any result similar to that in (i)?

7. $ABCD$ is a square of side length 1. Take $P$ to be any (general) point inside the square and draw a line through $p$ parallel to $AD$ meeting $AB$ and $DC$ at $Q, R$ respectively. Similarly draw a line through $P$ parallel to $AB$ meeting $AD$ and $BC$ at $T$ and $S$.

   Show that if the rectangle $AQPT$ has area larger than $\frac{1}{4}$ then the rectangle $PSCR$ has area less than $\frac{1}{4}$.

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\[1\text{Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.}\]