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Never Stand Still | Faculty of Science | School of Mathematics and Statistics

Solution Sheet 11, August 2, 2012

Answers

1. There are infinitely many solutions: all possible values for x, so long as for each x, y is chosen to be equal to x.

2. $45 \times \frac{6}{5} = 54$

3. (a) Easy

   (b) Using the equations, $(2^3 - 1) = 7$ and $(2^5 - 1) = 31$ are both factors, and $\frac{2^{15}-1}{7*31} = 151$ is the other prime factor.

   (c) Since

$$\begin{aligned} x^{15} + 1 &= (x^3+1)(x^{12} - x^9 + x^6 - x^3 + 1) \\ &= (x^5+1)(x^{10} - x^5 + 1). \end{aligned}$$

   then both $x^3 + 1 = 9$ and $2^5 + 1 = 33$ are factors of $x^{15} + 1$, and $\frac{2^{15}+1}{99} = 331$ is the other prime factor.

4. By Heron's Formula, the area of triangle APB is

$$\sqrt{s * (s - AP) * (s - PB) * (s - AB)} = 30\sqrt{6}$$

   where s is half the perimeter of APB. Hence the perpendicular distance of P from AB is $4\sqrt{6}$. Construct a right-angled triangle APQ where Q is on AB and $\angle AQP = 90$. Pythagoras' theorem tells us the distance $AQ = 10$. Hence by Pythagoras' theorem again $DP = \sqrt{10^2 + (4\sqrt{6})^2} = \sqrt{196} = 14$

5. Since $3m - 1$ is a multiple of n: $3n \geq 3m > 3m - 1 = kn$ for some k. This means that k can only be 1 or 2. When $k = 1$, $3m - 1 = n$ which, in conjunction with $3n - 1 = jm$ for some j, yields values of $(m, n) = (4, 11), (2, 5)$ or $(1, 2)$. Similarly when $k = 2$ we get $(m, n) = (5, 7)$ or $(1, 1)$.