

Solution Sheet 12, August 2, 2012

## Answers

1. For the numbers between 1000 and 1999, fix the most significant digit. If we count just the number of ways we can have double zero and one other digit, then we get 27 possibilities. Similarly for double 2, 3, 4,  $\dots$  9.

Counting the number of ways we can have two 1's is different, since we already have a one. If we count the number of ways we can have exactly one 1 and two other digits, then we get 297 possibilities.

Some of these numbers have already been counted (1100 for example), so we must take away  $3 * 9 = 27$  possibilities that have been double counted.

Brute-force counting tells us there are 11 possibilities between 2000 and 2012.

Hence there are  $8 * 27 + 297 - 27 + 11 = 497$  such numbers.

2.  $\frac{1}{100}$

3. (a)

$$\frac{1}{3 - \sqrt{7}} = -\frac{1}{6} - \frac{\sqrt{7}}{18}$$

- (b)

$$\frac{1}{3 - \sqrt[3]{7}} \frac{9 + 3\sqrt[3]{7} + \sqrt[3]{49}}{9 + 3\sqrt[3]{7} + \sqrt[3]{49}} = \frac{9}{20} + \frac{3\sqrt[3]{7}}{20} + \frac{\sqrt[3]{49}}{20}$$

4. (a) easy

(b)  $(a, b, c) = (3, 11, 13), (1, 11, 17), (5, 7, 15), (1, 7, 26), (5, 5, 19), (3, 5, 25), (1, 5, 35), (1, 3, 53), (1, 1, 10)$