## Solution Sheet 16, September 10, 2012

## Answers

1. 3 boxes of apples, 1 box of oranges, 4 boxes of lemons.
2. Find the area of the larger square in two different ways.
3. Let the rectangle have sides of length $a, b$ where

$$
\begin{equation*}
a+b=10 \tag{1}
\end{equation*}
$$

By Pythagoras' theorem, the diagonal $d$ has length

$$
\begin{equation*}
d^{2}=a^{2}+b^{2} \tag{2}
\end{equation*}
$$

Sub equation ?? into equation 2 , the result is

$$
d^{2}=2 a^{2}-20 a+100
$$

Use the formula for the minimum value of a parabola to find that $a=b=5$. Hence $d=2 \sqrt{5}$.
4. $y_{n}=1(\bmod 8)$ but $x_{2}=3(\bmod 3)$ and $x_{3}=5(\bmod 8)$. For larger values of $n$ : $x_{n}=3+2 \times 5(\bmod 8)=5(\bmod 8)$ or $x_{n}=5+2 \times 3(\bmod 8)=5(\bmod 8)$
5. Using $a^{2}+b^{2} \geq 2 a b$,

$$
\begin{aligned}
2\left(a^{2}+b^{2}+c^{2}\right) & \geq 2(a b+b c+c a) \\
3\left(a^{2}+b^{2}+c^{2}\right) & \geq\left(a^{2}+b^{2}+c^{2}\right)+2(a b+b c+c a) \\
3\left(a^{2}+b^{2}+c^{2}\right) & \geq(a+b+c)^{2} \\
\frac{a^{2}+b^{2}+c^{2}}{3} & \geq\left(\frac{a+b+c}{3}\right)^{2}
\end{aligned}
$$

