Errata: Solution Sheet 4, May 25, 2012

Answers

1. $f(4) = 6$. Generally, $f(n) = (n - 1)!$

2. $(22) \times (35) = 770$ is the only combination where both numbers are over 18.

3. (a) $50 + 20 - 10 = 60$. This is the number of numbers with a factor in common with 100. So the answer should be $100 - 60 = 40$.

   (b) $25 \times 102 + 10 \times 105 - 5 \times 110 = 3050$. This is the sum of numbers with a factor in common with 100. So the answer should be $1 + 2 + \cdots + 100 - 3050 = 2000$

4. $x = 0, 11$ are the 'fixed points’ (solve $x_{n+1} = x_n$), but there are many ways to get to either 0 or 11. Rewrite the recurrence relation so that $x_{n+1} = (x_n - 5)^2 - 25$. Then you can write $x_{n+1}$ in terms of $x_0$:

   $$x_{n+1} = ( ((x_0 - 5)^2 - 30)^2 - \cdots - 30 ) - 25$$

   Where there are $n - 1$ 30’s. So if $x_{n+1} = 0$, re-arrangement gives

   $$x_0 = 5 \pm \sqrt{30 + \sqrt{30 + \cdots \sqrt{30 \pm 5}}}$$

   Similarly for $x_{n+1} = 11$

   $$x_0 = 5 \pm \sqrt{30 + \sqrt{30 + \cdots \sqrt{30 \pm 6}}}$$

5. $x = y = z$, so solve $x + x^2 = 2$. 