Solution Sheet 8, June 20, 2012

Answers

1. Use the fact that \(1997^4\) ends in a one. Answer is 7

2. 500

3. Pythagoras’ Theorem.

4. Assume \(x \leq y\), then \((7, 42), (8, 24), (9, 18), (10, 15), (12, 12)\). Repeat for \(x\) and \(y\) swapped.

5. (a) 1

   (b) The sum of the geometric series \(S = 1 - 2 + 4 - 8 + \ldots + (-2)^{n-1} = \frac{1-(-2)^n}{1-(-2)} = \frac{1+2^n}{3}\) since \(n\) is odd. Then \(3S = 1 + 2^n\), so \(1 + 2^n\) is divisible by 3. Similarly \(1 + 2^m\) is divisible by 3. Hence the gcd is at least 3.

6. ... its a rectangle.