THE UNIVERSITY OF NEW SOUTH WALES

Never Stand Still | Faculty of Science | School of Mathematics and Statistics

Solution Sheet 9, July 26, 2012

Answers

1. There are 49 ways, and even more methods of arriving at this answer. Perhaps the easiest is to use cases starting with using $0, 1$ or 2 50 coins.

2. (a) 11002222

   (b) $220200_3 = 2 * 3^5 + 2 * 3^4 + 0 * 3^3 + 2 * 3^2 + 0 * 3^1 + 0 * 3^0 = 666$

3. sub in $x = 0$ to find a_0; $x = 1$ to find $a_0 + a_1 + \cdots a_{18}$; a_1 and a_{16} can be found using the difference of two squares, but I'm not convinced this is the best solution.

4. 34cm.

5. Proof by induction. Trivially true for $n = 1$. Suppose true for $n = k-1$, and prove true for $n = k$. If k of our $k+1$ chosen numbers are from the range $\{1, \cdots, 2(k-1)\}$, then we are done. If not, then $2k-1, 2k$ must both have be chosen. Assume that we have not chosen k and, of our chosen numbers in the range $\{1, \cdots, 2(k-1)\}$, no two are divisors (in either case, we are done). If we were to additionally choose k, then by the inductive hypothesis some chosen number a must divide or multiply k. Multiplication is not possible, so a divides k and must also divide $2k$.

6.

7. Let AQ be of length x, PT of length y. So the area of $APQT$ is xy and the area of $PSCR$ is $(1-x)(1-y)$. If $xy > 1/4$, then $x > 1/(4y)$, and

$$\begin{aligned}(1-x) &< 1 - \frac{1}{4y} \\ (1-y)(1-x) &< (1-y)(1 - \frac{1}{4y}) \\ (1-y)(1-x) &< \frac{(1-y)(4y-1)}{4y})\end{aligned}$$

But $(1-y)(4y-1) < y$ for all values of y (verification left to the reader). So

$$(1-y)(1-x) < \frac{(1-y)(4y-1)}{4y}) < \frac{y}{4y} = \frac{1}{4}$$