1. Find all the positive integers such that \( ab = 3a + 3b \).

2. Let \( K \) be the circumcircle through the vertices of a rectangle with sides \( a \) and \( b \). On each side of the rectangle construct a semicircle. This will give four crescents formed between these semicircles and \( K \). What is the sum of the areas of the four crescents?

3. Suppose that two non-parallel straight lines \( k \) and \( \ell \) meet at a point \( P \) which is not on the page of my book. Construct a line which would (if \( P \) did lie on the page) bisect the angle between the lines and pass through \( P \).

4. Suppose the last digit of \( x^2 + xy + y^2 \) is zero, and \( x \) and \( y \) are positive integers. Prove that the last \textbf{two} digits of \( x^2 + xy + y^2 \) are both zero.

5. (a) Use your calculator to show that \((2!)^{\frac{1}{2}} < (3!)^{\frac{1}{3}} < (4!)^{\frac{1}{4}}\).

   (b) Prove that for every integer \( n > 0 \), \((n!)^{\frac{1}{n}} < ((n + 1)!)^{\frac{1}{n+1}}\).

6. (a) There are 128 coins of two different weights, 64 of each. How can one always find two different coins by performing no more than 7 weighings on a regular balance?

   (b) There are eight coins of two different weights, four of each. How can one always find two different coins by performing two weighings on a regular balance?

**Senior Questions**

1. The function \( f(x) = x^x \) has an inverse \( g(x) \) provided we restrict the domain of \( f \) to \( x \geq 1 \). Find a formula for the derivative of \( g(x) \) in terms of \( x \) and \( g(x) \).

2. The exponential function is defined as being the solution, \( y(x) \), to the differential equations

\[
y(x) = \frac{dy}{dx}, \quad \text{such that } y(0) = 1.
\]

Prove \( e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \).

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1Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.