

MATHEMATICS ENRICHMENT CLUB.¹

Problem Sheet 2, May 14, 2013

1. One bell rings every 10 minutes, while another rings every 12 minutes. If the bells have just rung together, after how many minutes will they next ring together?
2. The two digit number x has **digits** $a3$ and y has **digits** $3a$. If $6528 \times x = 8256 \times y$, find the digit a .
3. At a Maths Enrichment Club of 35 students, it was found that 21 play the piano, 10 play the violin, and 18 play the recorder. Everyone plays at least one instrument and it is known that 7 people play only the violin. Ann, Bill and Cathy play all three instruments. How many people only play the piano?
4. Suppose we expand $(3 + 2x + x^2)^{1998}$ to obtain $a_0 + a_1x + a_2x^2 + \dots$
 - (a) Find a_0 and a_1 .
 - (b) Find $a_0 + a_1 + \dots$
 - (c) Find $a_0 - a_1 + a_2 - a_3 + \dots$
5. Suppose that a triangle has sides a, b, c such that $a + b + c = 2$.
 - (a) Show that $(1 - a)(1 - b)(1 - c) > 0$.
 - (b) Show that $a^2 + b^2 + c^2 + 2abc < 2$.
6. Let $ABCD$ be any convex quadrilateral with perimeter p and area a .
 - (a) Show that $\frac{1}{2}p < AC + BD < p$.
 - (b) Show that $2a \leq AC \cdot BD$
 - (c) What can you say about the quadrilateral if the product of the diagonals $AC \cdot BD$ equals $2a$?
7. Three equal circles of radius r with centres O_1, O_2, O_3 meet at a common point P . If their other common points are ABC show that triangles ABC and O_1, O_2, O_3 are congruent.

¹Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.