MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 4, May 28, 2013

1. (a) Show that whatever base b is used, the number $(21)_b$ is never equal to twice $(12)_b$.
   (b) Find all the numbers and all bases $b \leq 12$ for which there exists a two digit number $(ac)_b$ which is twice the number obtained by reversing its digits.
   (c) Find all bases b and all numbers $n = (ac)_b$ such that $n = 2 \times (ac)_b$.

2. In how many ways is it possible to write 1000 as a sum of consecutive odd integers?

3. Draw a right triangle ABC with right-angle at C and the sides marked a, b, c as usual.
   (a) Draw the enlargement $A'B'C'$ of ABC by a factor of a.
   (b) On the same diagram draw the enlargement $A''B''C''$ of ABC by a factor of b, lining up $B'C'$ with $A''C''$, so that A', B' and B'' are collinear, and thus form a new triangle $A'''B'''C'''$.
   (c) Explain why the angle at C''' is a right angle.
   (d) What theorem have you just proven and why?

4. In the triangle ABC, it is given that $\angle ABC = 140°$. Let D be a point on AC and E a point on AB such that the three triangles AED, EDB and DBC are all isosceles, with their vertices at A, E and D respectively. Find all the angles of the triangle ABC.

5. Let K, L be points on the sides AB, AD respectively of the convex quadrilateral $ABCD$ such that $AK = \frac{1}{3}AB$ and $AL = \frac{1}{3}AD$. Similarly, M, N are points on CD, CB such that $CM = \frac{1}{3}CD$ and $CN = \frac{1}{3}CB$.
   (a) Prove that $KLMN$ is a parallelogram.
   (b) Find the ratio of the area of $KLMN$ to the area of $ABCD$.

6. **Year 11 Question.** Suppose that m and n are positive real numbers. Use trigonometry to find the the maximum value of
$$\frac{m+n}{\sqrt{m^2+n^2}}.$$

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.

Senior Questions

1. The hypotenuse of a right-angled triangle is 15 cm and the radius of the inscribed circle is 2cm. Find the perimeter of the triangle.

2. Suppose we place one of the numbers 1,2,3,...,2000 into each of 2000 boxes. Remove the two numbers a and b from any two boxes, chosen at random, and put their difference $a-b$ into one of the two boxes chosen and remove the empty box. Repeat the process until only one box remains. Show that the number in this box must be even.