MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 16, September 1, 2015[1]

1. A draw contains an unsorted collection of black, white, pink, and blue coloured socks. If socks are taken at random, one at a time, what is the minimum number which must be taken to be certain of finding five matching pairs?

2. Using each of the digits $1, 2, 3, 4$ and 5 exactly once to form 5-digit numbers, how many are divisible by 12?

3. In a wrestling tournament, there are 100 participants, all of different strengths. The stronger wrestler always wins over the weaker opponent. Each wrestler fights twice and those who win both of their fights are given awards. What is the least possible number of awardees?

4. Find the highest power of 2 that divides $33!$.

5. Let p be a prime number and x, y non-negative integers. Find all possible solutions to $p^x = y^4 + 4$.

6. Each of 11 weights is weighing an integer number of grams. No two weights are equal. It is known that if all these weights or any group of them are placed on a balance then the side with a larger number of weights is always heavier. Prove that at least one weight is heavier than 35 grams.

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*.

Senior Questions

1. Find all positive integers n for which the following statement holds:

   For any two polynomials $P(x)$ and $Q(x)$ of degree n there exist monomials ax^k and bx^l, $0 \leq k, l \leq n$, such that the graphs of $P(x) + ax^k$ and $Q(x) + bx^l$ have no common points.

2. A disc of radius 1 unit is cut into quadrants (identical quarters), and the quadrants are placed in a square of side 1 unit.

   (a) What is the least possible area of overlap?

   (b) What are the other possible areas of overlap?

3. There are five distinct real positive numbers. It is known that the total sum of their squares and the total sum of their pairwise products are equal.

   (a) Prove that we can choose three numbers such that it would not be possible to make a triangle with sides' lengths equal to these numbers.

   (b) Prove that the number of ways to form the triples satisfying (a) is at least six (triples which consist of the same numbers in different order are considered the same).