1. Find the number of ordered pairs \((x, y)\) of non-negative integers such that \(x + y \leq 100\).

2. Let \(p\) be your favourite prime number greater than 100, and \(a, b\) positive integers such that \(p^2 + a^2 = b^2\). Find \(\frac{a+b}{p}\).

3. A square is inscribed in a circle with diameter 2. Four smaller circles are then constructed with their diameters on each of the sides of the square; see below. Find the shaded area.

4. At a party of 21 people each person knows at most four others. Prove that there are five in the party who mutually do not know each other.

5. Let \(f(x)\) be a polynomial with integer coefficients. Suppose \(a_1, a_2, a_3, a_4, a_5\) are distinct integers such that \(f(a_1) = f(a_2) = f(a_3) = f(a_4) = f(a_5) = 2015\). Find the number of integral solutions for the equation \(f(x) = 2016\).

6. \(M\) is the midpoint of the side \(CA\) of triangle \(ABC\). \(P\) is some point on the side \(BC\). \(AP\) and \(BM\) intersect at the point \(O\). If \(BO = BP\), determine \(\frac{|OM|}{|PC|}\).

**Senior Questions**

1. Let \(P(x) = x^{100} + a_{99}x^{99} + a_{98}x^{98} + \ldots + a_2x^2 + a_1x + 1\) be a polynomial with all real roots, where \(a_1, a_2, \ldots, a_{99}\) are positive and real. Find the maximum value of the positive integer \(N\) in the inequality \(2(2^N - 1) \leq \sum_{i=1}^{99} a_i\).

2. Suppose that the integers \(x, y\) and \(z\) have greatest common divisor 1, and that \(\frac{1}{x} + \frac{1}{y} = \frac{1}{z}\). Show that \(x + y\) is a square.

3. Let \(n\) be a nonnegative integer. Prove that \(14^n + 11\) is never prime.

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1Some problems from UNSW’s publication *Parabola*, and the *Tournament of Towns in Toronto*. 