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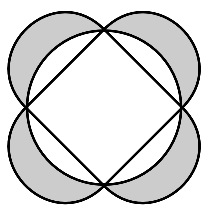
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 7, June 9, 2015[1]

1. Find the number of ordered pairs (x, y) of non-negative integers such that $x + y \leq 100$.

2. Let p be your favourite prime number greater than 100, and a, b positive integers such that $p^2 + a^2 = b^2$. Find $\frac{a+b}{p}$.

3. A square is inscribed in a circle with diameter 2. Four smaller circles are then constructed with their diameters on each of the sides of the square; see below. Find the shaded area.



4. At a party of 21 people each person knows at most four others. Prove that there are five in the party who mutually do not know each other.

5. Let $f(x)$ be a polynomial with integer coefficients. Suppose a_1, a_2, a_3, a_4, a_5 are distinct integers such that $f(a_1) = f(a_2) = f(a_3) = f(a_4) = f(a_5) = 2015$. Find the number of integral solutions for the equation $f(x) = 2016$.

6. M is the midpoint of the side CA of triangle ABC. P is some point on the side BC. AP and BM intersect at the point O. If $BO = BP$, determine $\frac{|OM|}{|PC|}$.

Senior Questions

1. Let $P(x) = x^{100} + a_{99}x^{99} + a_{98}x^{98} + \ldots + a_2x^2 + a_1x + 1$ be a polynomial with all real roots, where $a_1, a_2, \ldots, a_{99}$ are positive and real. Find the maximum value of the positive integer N in the inequality $2(2^N - 1) \leq \sum_{i=1}^{99} a_i$.

2. Suppose that the integers x, y and z have greatest common divisor 1, and that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$. Show that $x + y$ is a square.

3. Let n be a nonnegative integer. Prove that $14^n + 11$ is never prime.

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*.