1. How many integral solutions \((x, y)\) are there for the equation \(x^2 - y^2 = 1999\) (Note that 1999 is prime).

2. Let \(\triangle ABC\) be a right-angled triangle with sides of length \(|AB| = a\), \(|BC| = b\) and \(|CA| = c\) and \(\angle CAB = 90^\circ\). A circle is inscribed in \(\triangle ABC\) such that the circle intersects each side of the \(\triangle ABC\) exactly once. Find the radius of the circle in terms of \(a, b\) and \(c\).

3. Let \(N\) be a number of the form \(N = \underbrace{333...333}_{61\times3's}\), and \(M\) a number of the form \(M = \underbrace{666...666}_{62\times6's}\). Find \(N \times M\).

4. Let \(x\) be a positive odd number, and \(a\) a positive integer greater than 2. If \(a^x\) has remainder \(r_1\) when divided by \((a - 1)\) and \(r_2\) when divided by \((a + 1)\), find \(r_1 + r_2\).

5. Let \([x]\) denotes the greatest integer less than or equal to \(x\), where \(x\) is some real number. How many positive integers less than 1001 can be expressed in the form \([2x] + [4x] + [6x] + [8x]\)?

6. On a bicycle, tyre wear is proportional to distance traveled, front tyre lasting \(x\) kilometres and rear tyre lasting \(y\) kilometres. \((x < y)\). An advertisement claims that a set of tyres lasts at least \((x + y)/2\) kilometres provided you interchange front and rear tyre after an appropriate distance. Investigate.

\(^1\)Some problems from UNSW’s publication Parabola.
Senior Questions

1. Consider the quadratic equation

\[ f(x) = x^2 - 2(c + 1)x + c - 3, \]

where \( c \) is some real number. Let \( \alpha, \beta > 0 \), and suppose \( \alpha + \frac{1}{\alpha} \) and \( 2 - \beta - \frac{1}{\beta} \) are the roots of \( f(x) \). Find all possible values for \( c \).

2. Let \( \triangle ABC \) be a triangle and \( X, Y, Z \) points on the sides \( BC, CA, AB \) respectively. Suppose \( BX \leq XC, CY \leq YA, AZ \leq ZB \). Show that

(a) The area of \( \triangle XYZ \) is not less than one quarter of the area of \( \triangle ABC \).
(b) One of the corner triangles \( \triangle AZY, \triangle BXZ, \triangle CYZ \) has area not greater than the area of \( \triangle XYZ \).

![Diagram of triangle with points X, Y, Z on sides]

3. Given that \( a, b \) and \( c \) are positive integers, find the conditions for which the equation

\[ \sqrt{a} - b = \sqrt{c} \]

has a solution.

4. (bonus) Infinitely many physicists walks into a pub. The first physicist orders a beer, the second orders half a beer, the third a quarter, the fourth an \( 8^{th} \) and so on. The bartender happens to be a math student, what would the bartender tell the physicists?