MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 9, June 23, 2015[1]

1. Given that x and y are integer, how many different solutions does the equation $|x| + 2|y| = 100$ have?

2. Place the numbers $1, 2, 4, 8, 16, 32, 64, 128$ and 256 in a 3×3 square grid in such a way, that the product of each row, column and diagonal gives the same value. Counting different orientations of the grid as the same solution, can you find other solutions?

3. Brackets are to be inserted into the expression $10 \div 9 \div 8 \div 7 \div 6 \div 5 \div 4 \div 3 \div 2 \div 1$ so that the result is an integer.

   (a) Determine the maximum value of this integer.

   (b) Determine the minimum value of this integer.

4. If x is a positive real number, let $[x]$ denotes the greatest integer less than or equal to x and $\{x\} = x - [x]$. For example, $[3.14] = 3$, $\{3.14\} = 0.14$.

   Find numbers x, y such that

   (a) $x^3 - 5[x] = 10$.

   (b) $y^3 - 5\{y\} = 10$.

5. Consider the polynomial $f(x) = x^4 - nx + 63$. Find the smallest positive integer n such that $f(x)$ can be written as the product of two non-constant polynomials with integer coefficients.

6. Find the last three digits of

$$2015^{2014^{2013^{\cdot^{\cdot^{1}}}}}.$$

[1] Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*.

Senior Questions

1. How many different integers x satisfy the equation

$$(x^2 - 5x + 5)^{x^2 - 11x + 30} = 1$$

2. $5, 11, 17, 23$ and 29 are given prime numbers in arithmetic progression. Find *six* prime numbers in arithmetic progression.

3. Given that n is a positive integer and $2n + 1$ and $3n + 1$ are perfect squares, prove that n is divisible by 40.