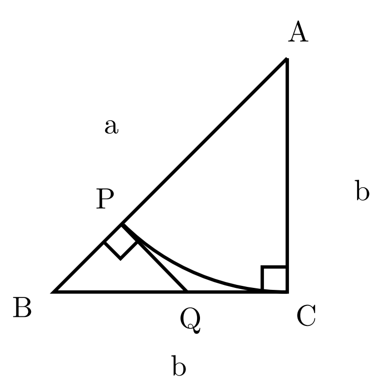
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 1, April 28, 2015

1. Express $0.284284284\ldots$ as a fraction in lowest terms.

2. For any whole number n, use the fact that
$$\frac{n-1}{n} - \frac{n-2}{n-1} = \frac{1}{n \times (n-1)},$$
to calculate
$$1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{10100}.$$

3. A sports club has a total of 163 members. The club offers the choice of basketball, cricket and soccer. Each member selects one or more activity to do: 73 plays cricket, 100 plays basketball, 60 plays atleast two different sports and 10 plays all three.
   (a) How many members plays soccer?
   (b) There are 25 members that plays both basketball and cricket, how many members plays soccer *only*?

4. ABC is a right-angled isosceles triangle. Assuming $a > b$, the edge AB has length a, and AC, BC has equal length b. The points P is made by swinging AC in a circular arc onto AB, and the points Q is the interception of perpendicular line from P to BC; as shown below.



(a) Find the length of QC.

(b) Let a and b be whole numbers, show that the ratio $a/b \neq \sqrt{2}$.

5. (a) Show that if a whole number is divisible by 4, then so is the number formed by its last two digits.

   (b) Show that if a whole number is divisible by 9, then so is the sum of all of its digits.

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6. $[n(n+1)(n+2)]^2 = 481273563 * 6$, use the results of 5. to find the missing digit $*$.

Senior Questions

1. Prove the identity

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}.$$

2. Using the above result, show that the infinite series satisfies

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots = \tan^{-1}(x).$$

3. For an integer n, show that $n(n+1)(n+2)(n+3)+1$ is a perfect square. Thus evaluate $\sqrt{(31)(30)(29)(28)+1}$.