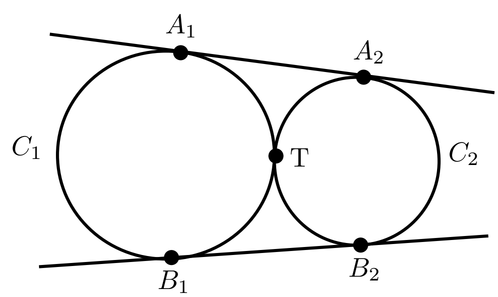
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 3, May 12, 2015[1]

1. The diagonal of a polygon is a line joining two non-adjacent vertices. For example, the two diagonals of a square. A polygon has 152 diagonals, how many vertices does it have?

2. Find all primes p such that $17p + 1$ is a square.

3. A politician counted that the total number of staff working for him is the product of two consecutive numbers. In an upcoming election campaign, the politician plans to visit the schools in a city. The politician decides to split all his staff, himself and his wife into groups of three to make the visits, explain to the politician why this is mathematically impossible.

4. Two circles C_1 and C_2 with centers O_1 and O_2, are externally tangent to each other at T. Their common tangent meets at A_1, A_2 and B_1, B_2 respectively. Prove that the circles with diameters A_1A_2 and B_1B_2 are tangent to each other at T.



5. Each of six baskets contains some mangoes, peaches and apples. The number of peaches in each basket is equal to the total number of apples in the other five baskets, and the number of apples in each basket is equal to the total number of mangoes in the other give baskets. Prove that the total number of fruit in the six baskets is a multiple of 31

6. On a race track are 33 cyclist, riding in the same direction, each at a different constant speed. There is only one point along the track at which a cyclist is allowed to pass another cyclist; the race is stopped if any cyclist attempts to make a pass at any other point along the track. Can they continue to ride for an arbitrarily long period?

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*

Senior Questions

1. Alex has a piece of cheese. He chooses a positive number $x > 1$ and cut the piece into two, in the ratio $1 : x$. He can then choose any piece and cut it in the same way. Is it possible for him to obtain, after a finite number of cuts, two piles of pieces each containing half the original amount of cheese?

2. Find a polynomial function $f(x)$ which has its inverse equal to it's derivative;

$$f^{-1}(x) = f'(x).$$

3. (a) Find the next 3 terms of the sequence $1, 1, 2, 3, 5, 8, 13, \ldots$

   (b) Let F_n be the n^{th} term of the above sequence. Prove the sum of the sequence is $S_n = F_{n+2} - 1$.