MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 12, August 8, 2016

1. Find the smallest possible integer \( n \), such that \( n + 2n + 3n + \ldots + 99n \) is a perfect square.

2. Let

\[
f(n) = \frac{1 + 2 + 3 + \ldots + n}{n}.
\]

Evaluate \( f(1) + f(2) + f(3) + \ldots + f(99) + f(100) \).

3. \( P \) is a point inside a convex polygon whose sides are all equal in length. Perpendiculars are constructed from \( P \) to the sides of the polygon. Show that the sum of the lengths of the perpendiculars is the same for all positions of \( P \).

4. Let \( A \), \( B \) and \( C \) be integers. Find the smallest possible prime \( p \), such that

\[
\frac{x^2 - p}{(x-2)(x-3)(x-5)} = \frac{A}{x-2} + \frac{B}{x-3} + \frac{C}{x-5}.
\]

5. Is it possible to make a 4 \( \times \) 4 square lattice of size 4 cm by 4 cm by using

(a) 5 pieces of thread, each 8 cm long?
(b) 8 pieces of thread, each 5 cm long?

![Diagram of a 4 \( \times \) 4 square lattice]

6. Find the last two digits of \( \sqrt{4^{2016} + 2 \times 6^{2016} + 9^{2016}} \).
Senior Questions

1. Given 2 three digit numbers $a$ and $b$ and a four digit number $c$. If the sum of the digits of the number $a + b$, $b + c$ and $c + a$ are all equal to 3, find the largest possible sum of the digits of the number $a + b + c$.

2. Are there integers $a, b$ which satisfy

$$5a^2 - 7b^2 = 9?$$

Either find them or show that they do not exist.

3. Prove that there is no convex eight sided polygon with all angles equal and the sides distinct integers.