MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 12, August 8, 2016

1. Find the smallest possible integer n, such that $n + 2n + 3n + \ldots + 99n$ is a perfect square.

2. Let
$$f(n) = \frac{1 + 2 + 3 + \ldots + n}{n}.$$
Evaluate $f(1) + f(2) + f(3) + \ldots + f(99) + f(100)$.

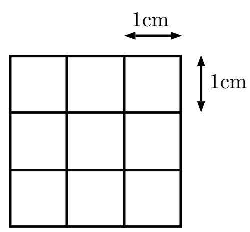
3. P is a point inside a convex polygon whose sides are all equal in length. Perpendiculars are constructed from P to the sides of the polygon. Show that the sum of the lengths of the perpendiculars is the same for all positions of P.

4. Let A, B and C be integers. Find the smallest possible prime p, such that
$$\frac{x^2 - p}{(x-2)(x-3)(x-5)} = \frac{A}{x-2} + \frac{B}{x-3} + \frac{C}{x-5}.$$

5. Is is possible to make a 4×4 square lattice of size 4 cm by 4 cm by using

   (a) 5 pieces of thread, each 8 cm long?

   (b) 8 pieces of thread, each 5 cm long?



6. Find the last two digits of $\sqrt{4^{2016} + 2 \times 6^{2016} + 9^{2016}}$.

Senior Questions

1. Given 2 three digit numbers a and b and a four digit number c. If the sum of the digits of the number $a+b$, $b+c$ and $c+a$ are all equal to 3, find the largest possible sum of the digits of the number $a+b+c$.

2. Are there integers a, b which satisfy

$$5a^2 - 7b^2 = 9?$$

   Either find them or show that they do not exist.

3. Prove that there is no convex eight sided polygon with all angles equal and the sides distinct integers.