MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 8, June 20, 2016

1. Alex has 12 Tim-Tams of which 9 are original and 3 are the white type. Alex wants to share her Tim-Tams with 3 friends. She puts the Tim-Tams in a box and she and her 3 friends each randomly choose 3 Tim-Tams.

   What is the probability that exactly one of them ends up with all 3 white Tim-Tams.

2. On a blackboard, there are 2016 integers, from 1 to 2016 (including 1 and 2016). You are allowed to remove two integers you like, but you have to add the arithmetic mean as a new number. (If you delete for example 10 and 11, you have to write 10.5 as the new number). Therefore, the number of integers on the board decrease in every turn, and at the end of the game, only one integer will remain. What is the lowest value of this number?

3. Prove that every term in the infinite sequence $18, 108, 1008, 10008, 100008, ...$, is divisible by 18.

4. How many factors of $40! := 40 \times 39 \times 38 \times \ldots \times 2 \times 1$ are perfect cubes?

5. Are the following statements true or false? Prove your answers.

   (a) A pentagon inscribed in a circle and having all of its angles equal *must* have all of its sides equal.

   (b) A hexagon inscribed in a circle and having all of its angles equal *must* have all of its sides equal.

6. A square billiard table with side length 1 metre has a pocket at each corner. A ball is struck from one corner and hits the opposite wall at a distance of $\frac{19}{96}$ metres from the adjacent corner. If the ball keeps travelling, how many walls will it hit before it falls into a pocket?

Senior Questions

1. Arrange the digits from 1 to 9 in such an order that the first two digits form a number that is a multiple of 2, the first three digits form a number that is a multiple of 3, and so on.

   Example: 123456789 doesn't work, because 12 is divisible by 2 and 123 is divisible by 3, but 1234 is not divisible by 4.

2. Let a, b be positive integers and p prime. Solve

$$(b - a) = \frac{2ap^2}{a^2 - p^2}.$$

3. Find a sixth-degree polynomial with integer coefficients which is a factor of $x^{15} + 1$.