



**MATHEMATICS ENRICHMENT CLUB.**

**Problem Sheet 2, May 14, 2018**

1. What is the smallest number divisible by 1, 2, 3, ..., 10?
2. Suppose  $n$  is an integer greater than 1.
  - (a) Show that  $n^2 + n$  lies strictly between two squares and so cannot be a square.
  - (b) Use the same idea to show that  $n^4 + n^3 + n^2 + n$  is not a perfect square. (Note that it is when  $n = 1$ .)
3. Find the side length of the square whose base lies on the base of an isosceles triangle with sides 10, 10 and 12, with two vertices touching the equal sides of the triangle.
4. Find the largest positive integer that when divided into each of 364, 414 and 539 leave the same remainder.
5. Let  $AB$  be a chord of a circle centre  $O$  and let  $P$  be a point on its circumference. If  $\angle APB = \angle AOB$ , find this angle.
6. There are 128 coins of two different weights, 64 of each. How can one always find two different coins by performing no more than 7 weighings on a regular balance?

**Senior Questions**

1. It can be shown that the sum to infinity of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}.$$

Use this to find the sum to infinity of the series

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$$

and

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

2. Robin Hood's apprentice still has a lot to learn when it comes to archery. She can always hit the target, but is equally likely to hit any point on the target. On a square target, what is the probability that she will land the arrow closer to the bullseye (the centre) than an edge?