MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 4, May 28, 2018

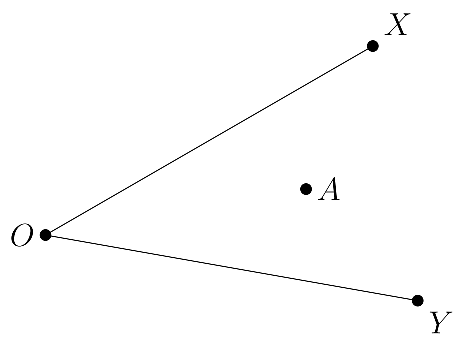
1. Find the number of solutions to the equation

$$x^2y^3 = 6^{12},$$

where x and y are positive integers.
(AMC 2006 Intermediate Division Q2)

2. $\dfrac{1}{a+\frac{1}{b+\frac{1}{c+\frac{1}{d}}}} = \dfrac{11}{42}$, where a, b, c and d are positive integers. Find $a+b+c+d$.
(AMC 2006 Intermediate Division Q1)

3. As shown in the diagram, $\angle XOY$ is acute and A is a point lying inside this angle.



Find a point B on the side OX and a point C on the side OY such that the perieter of the triangle ABC is minimised.
(Adapted from Kiselev's Geometry Book 1: Planimetry)

4. What is the sum of all the digits used in writing down the numbers from one to 9999?

Senior Questions

1. $x^2 - 19x + 94$ is a perfect square and x is an integer. What is the largest value of x? (AMOC 2007 Intermediate paper)

2. This is the first part of Question Sixteen from the 2017 HSC Mathematics paper.

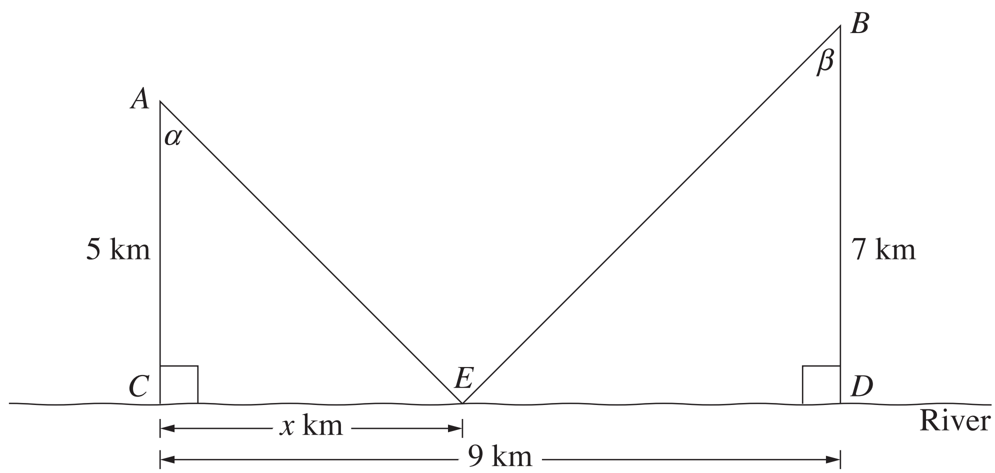
(a) John's home is at point A and his school is at point B. A straight river runs nearby.

The point on the river closest to A is point C, which is 5 km from A.

The point on the river closest to B is point D, which is 7 km from B.

The distance from C to D is 9 km.

To get some exercise, John cycles from home directly to point E on the river, x km from C, before cycling directly to school at B, as shown in the diagram.



The total distance John cycles from home to school is L km.

(i) Show that $L = \sqrt{x^2 + 25} + \sqrt{49 + (9 - x)^2}$. **1**

(ii) Show that if $\dfrac{dL}{dx} = 0$, then $\sin\alpha = \sin\beta$. **3**

(iii) Find the value of x that makes $\sin\alpha = \sin\beta$. **2**

Find a more elegant way (that is, one that does not use calculus) to solve the max-min problem in Question Sixteen.