1. Find the number of solutions to the equation
\[ \begin{align*}
x^2y^3 &= 6^{12},
\end{align*} \]
where \( x \) and \( y \) are positive integers.
(AMC 2006 Intermediate Division Q2)

2. \[ \begin{align*}
a + b + c + d &= \frac{11}{42},
\end{align*} \]
where \( a, b, c \) and \( d \) are positive integers. Find \( a + b + c + d \).
(AMC 2006 Intermediate Division Q1)

3. As shown in the diagram, \( \angle XOY \) is acute and \( A \) is a point lying inside this angle.

\[
\begin{array}{c}
\text{Find a point } B \text{ on the side } OX \text{ and a point } C \text{ on the side } OY \text{ such that the perimeter of the triangle } ABC \text{ is minimised.}
\end{array}
\]
(Adapted from Kiselev’s Geometry Book 1: Planimetry)

4. What is the sum of all the digits used in writing down the numbers from one to 9999?
Senior Questions

1. $x^2 - 19x + 94$ is a perfect square and $x$ is an integer. What is the largest value of $x$? (AMOC 2007 Intermediate paper)

2. This is the first part of Question Sixteen from the 2017 HSC Mathematics paper.

(a) John’s home is at point $A$ and his school is at point $B$. A straight river runs nearby.

The point on the river closest to $A$ is point $C$, which is 5 km from $A$.

The point on the river closest to $B$ is point $D$, which is 7 km from $B$.

The distance from $C$ to $D$ is 9 km.

To get some exercise, John cycles from home directly to point $E$ on the river, $x$ km from $C$, before cycling directly to school at $B$, as shown in the diagram.

![Diagram of river and points](image)

The total distance John cycles from home to school is $L$ km.

(i) Show that $L = \sqrt{x^2 + 25} + \sqrt{49 + (9 - x)^2}$. 1

(ii) Show that if $\frac{dL}{dx} = 0$, then $\sin \alpha = \sin \beta$. 3

(iii) Find the value of $x$ that makes $\sin \alpha = \sin \beta$. 2

Find a more elegant way (that is, one that does not use calculus) to solve the max-min problem in Question Sixteen.