UNSW AUSTRALIA

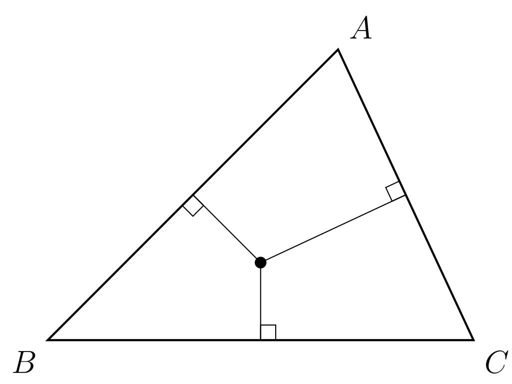
Never Stand Still | Science | School of Mathematics and Statistics

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 5, June 4, 2018

1. If a and b are positive integers with $a > b$, and $(a+b)^2-(a-b)^2 > 29$, find the smallest possible value of a.

2. If the straight line $y = x + c$ meets the circle $x^2 + y^2 = 1$ at a single point, find the value(s) of c.

3. Let ABC be a triangle. Prove that the perpendicular bisectors of the sides AB, AC and BC intersect at a single point. (This point is called the circumcentre of the triangle.)



4. Without using a calculator, show that

$$\sqrt[3]{5\sqrt{13}+18}-\sqrt[3]{5\sqrt{13}-18}=3.$$

Hint: Let $x = a - b$ and then cube.

5. If x and y are positive integers which satisfy $x^2-8x-1001y^2 = 0$, what is the smallest possible value of $x + y$?
(AMC 2012 Senior Division Q23)

Senior Questions

1. Suppose that $g(x)$ is an odd function. Show that, if g is defined at $x = 0$, then $g(0) = 0$.

2. (a) Suppose that $f(x)$ is an even function defined for all real x and differentiable throughout its domain. Show that $f'(x)$ is an odd function.

   (b) Similarly, suppose that $g(x)$ is an odd function defined for all real x and differentiable throughout its domain. Show that $g'(x)$ is an even function.

3. Suppose that $h(x)$ is defined for all real x. Then $h(x)$ can be written as

$$h(x) = f(x) + g(x),$$

   where f is an even function and g is an odd function. Explain how to do this.

4. Is there a function, defined for all real x, that is both odd and even?